Physics in Medicine:
Fundamentals of Analyzing Biomedical Signals

Klaus Lehnertz

Topics:

- theory of nonlinear dynamical systems
- characterizing measures
- biosignals and recording of biosignals
- applications (medicine, physics, biology)

http://epileptologie-bonn.de/cms/front_content.php?idcat=203

Literature:

Physics:

- E. Ott: *Chaos in dynamical systems*. 2nd ed., Cambridge University Press, Cambridge UK (2002)
- **H. Kantz, T. Schreiber: *Nonlinear time series analysis*. 2nd ed., Cambridge University Press, Cambridge (2003)**
- **A. Pikovsky, M. Rosenblum, J. Kurths: Synchronization: A universal concept in nonlinear science, Cambridge University Press, Cambridge (2001)**

Medicine:

- E. Basar, T.H. Bullock: *Brain Dynamics*. Series in Brain Dynamics Vol. 2, Springer, Berlin (1989)
- E. Basar: *Chaos in Brain Function*. Springer, Berlin (1990)
- E.R. Kandel, J.H. Schwartz, T.M. Jessell: *Principles of Neural Science*. 4th ed., Elsevier North Holland, New York (2000)

Literature:

Signal processing, Statistics, Computing:

- A.V. Oppenheim; A.S. Willsky: *Signals and systems*. Prentice Hall, 1996
- J.S. Bendat, A.G. Piersol: *Random Data: Analysis and measurement procedures*. 4th ed., Wiley Interscience, New York, 2010

- B.R. Martin: *Statistics for Physicists*. Academic Press, London, New York, 1971
- P.R. Bevington: *Data reduction and error analysis for the physical sciences*. McGraw-Hill, New York, 2002.

- W.H. Press, B.P. Flannery, S.A. Teukolsky, W.T. Vetterling: *Numerical Recipes. The art of scientific computing*. 3rd ed., Cambridge University Press, Cambridge, 2007

TISEAN: Nonlinear Time Series Analysis https://www.pks.mpg.de/~tisean/

TISEAN

Nonlinear Time Series Analysis

Rainer Hegger
Holger Kantz
Thomas Schreiber

Go to Version 3.0.1 (released March 2007)

Go to Version 2.1 (released December 2000)

pytisean on GitHub

Historical overview:




1778 P. Laplace (Laplace’s demon, “everything is predictable”)

1880 W. Sierpinski (non-Euclidian geometry, “mathematical monster”)

1892 H. Poincare (three-body problem, dimension of manifolds)

1919 F. Hausdorff (extension of notion of dimension)

1963 E. Lorenz (weather forecasting)

1967 B. Mandelbrot (fractals, self similarity)

1975 J. Yorke (deterministic chaos)

1977/78 routes into chaos:

S. Grossmann/S. Thomae (period doubling)

M. Feigenbaum (Feigenbaum constant),

Newhouse-Ruelle-Takens route

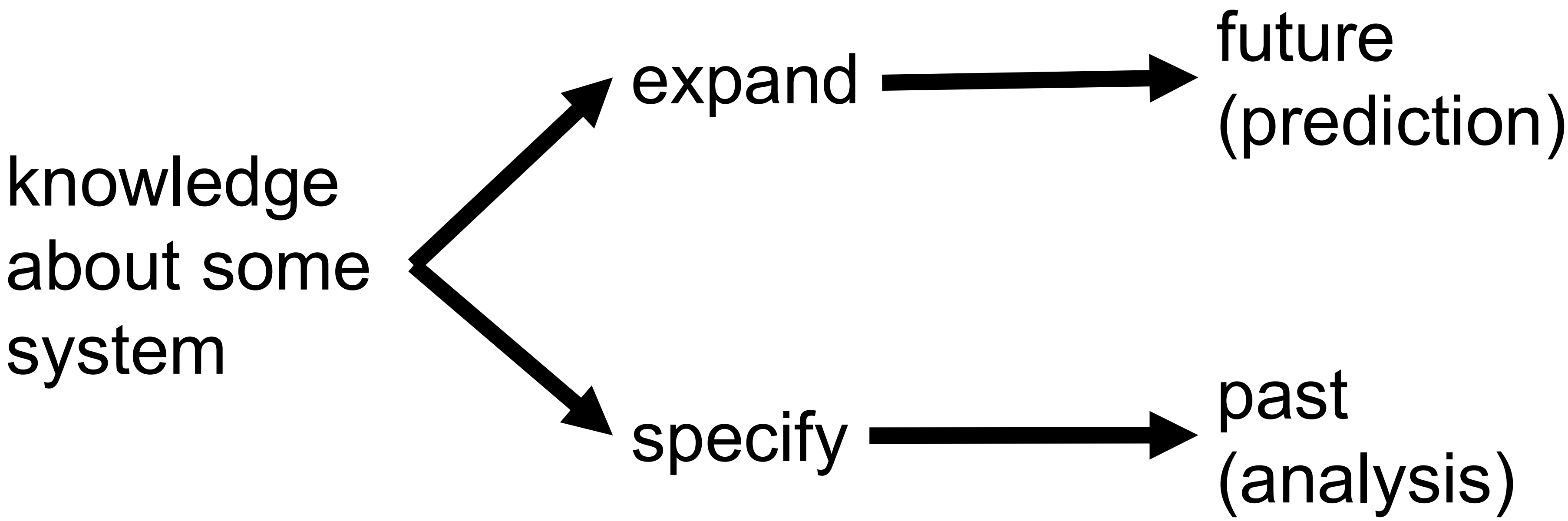
1981 D. Ruelle (strange attactors),

P. Grassberger / I. Procaccia (correlation dimension)

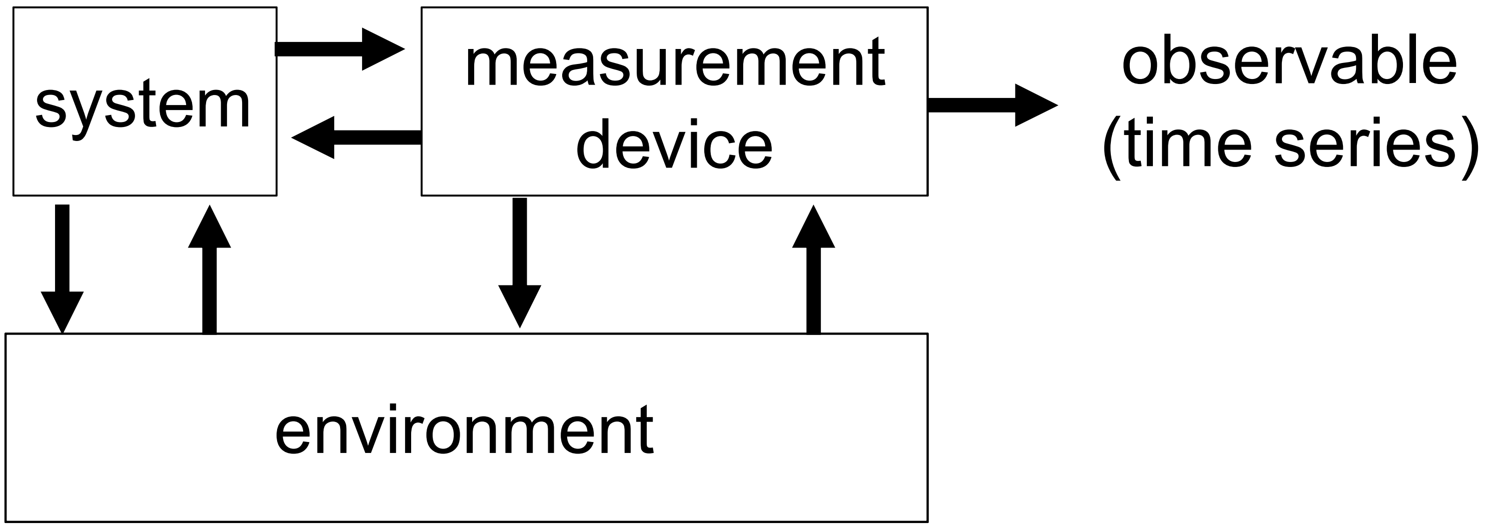
F. Takens (state space reconstruction)

since 1990 nonlinear time series analysis

Time series analysis **Goals**



Time series analysis — Measurement



note: interactions !

- what is a suitable device ?
- what is a good observable ?
- what is a suitable environment ?
- interfaces ?

Time series analysis **Time series**

time series: - sequence of data (length N)
- measurement or simulation (model)
- time-dependent

$$\left(v_i, v_{i+\Delta t}, \ldots, v_{N\Delta t}\right)$$

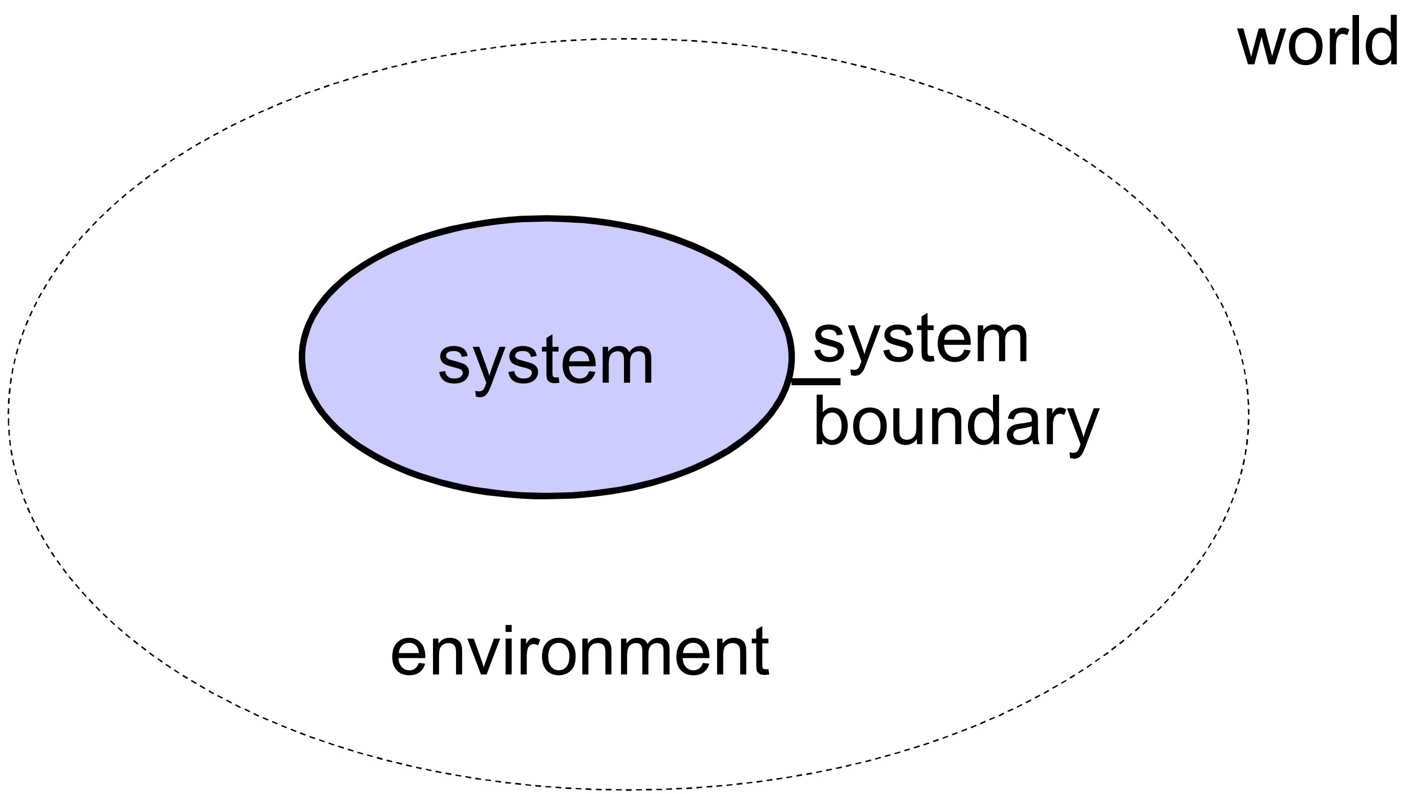
Δt - temporal distance between successive data points
- sampling interval (measurement)

Time series analysis

Time series

	experiment	model simulation
length of time series	(mostly) limited	user-defined
sampling interval	limited	user-defined
precision	A/D converter noise	user-defined

System



interactions: system – environment?

open system?

isolated system?

Dynamical system

- system under influence of some force ($\delta\upsilon\nu\alpha\mu\iota o$ = force)
- **time-dependent** system states
- state changes depend on current state

deterministic	**stochastic**
same initial states	same initial states
⬇	⬇
same evolution	random evolution

Dynamical system

- characterized by time-dependent state variables $\mathbf{x}(t) \in \mathbb{R}^d$
- temporal evolution of state variables:

continuous case: set of (first-order) ordinary differential equations with initial conditions $\mathbf{x}(0) = \mathbf{x}(t_0)$

$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = f(t, \mathbf{x}(t), \beta)$$

discrete case: set of difference equations (mapping) with initial conditions $\mathbf{x}_0 = \mathbf{x}_{t_0}$

$$\mathbf{x}_{t+\Delta t} = F(T, \mathbf{x}_t, \beta)$$

with d = dimension of system; β = control parameter;
f, F = nonlinear functions in case of nonlinear systems

Nonlinearity - Linearity

linear — simple

equations can have — solutions

nonlinear — complex

---

impact of changing control parameters or initial conditions?

system		**effect**
linear	→	proportional
nonlinear	→	non-proportional

physics

chemistry

biology

geosciences

nonlinear dynamics

medicine

engineering

economics

politics/ military

cultural sciences

nonlinear dynamics in physics

condensed matter physics

pattern formation
phase transitions
spin waves

mechanics

nonlinear oscillators
coupled/forced pendulums
magneto-mechanic oscillators
torsion bar

acoustics

sound generation with:
- laser
- musical instruments

fluid mechanics

transition to turbulent motion
crystal growth
surface of liquids

astrophysics

solar system
motion of stars
sun spots
pulsar/quasar
distribution of galaxies

laser physics

laser instabilities
semiconductor laser
coupled laser

optics

opto-galvanic systems
nonlinear optics

plasma physics

oscillations in gas discharges
pattern formation
plasma waves

nonlinear dynamics in medicine/ biology

physiology

biochemistry

neurology
epileptology
psychiatry

biological oscillations

epidemiology

psychology

ecology

endocrinology

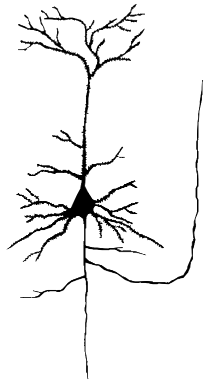
population dynamics

pneumology

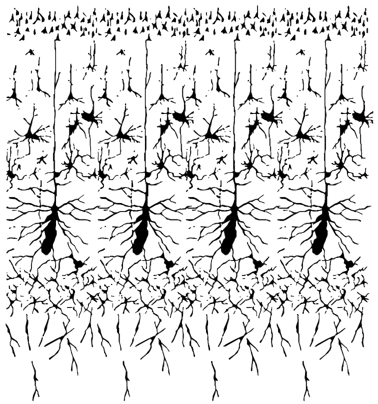
genetics

cardiology

nephrology



neuron:
highly
nonlinear



neuron pools:
networks with complex
interactions
electromagnetic, chemical, morphological

nonlinear (?), open,
dissipative, adaptive

approx. 10^{11} neurons
each with
10^{3} - 10^{4} synapses

Nonlinearity and Causality

physical phenomenon → reason? → strongly deterministic causality principle

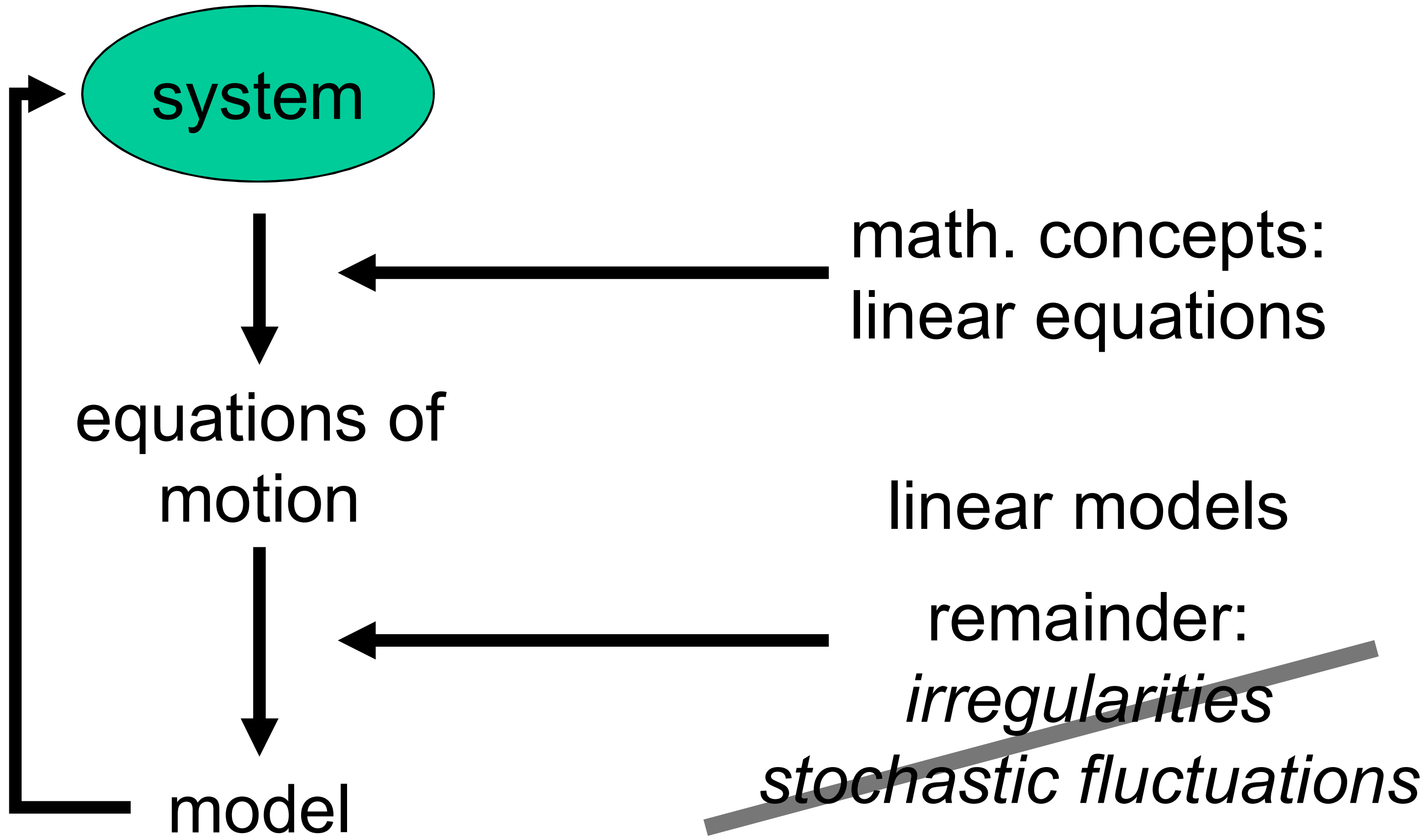
(exactly) equal causes have (exactly) equal effects

past — now — future → time

math. model (ODE)
+
initial condition

system predictable
+
behavior reproducible

The pragmatic perspective of a linear nature



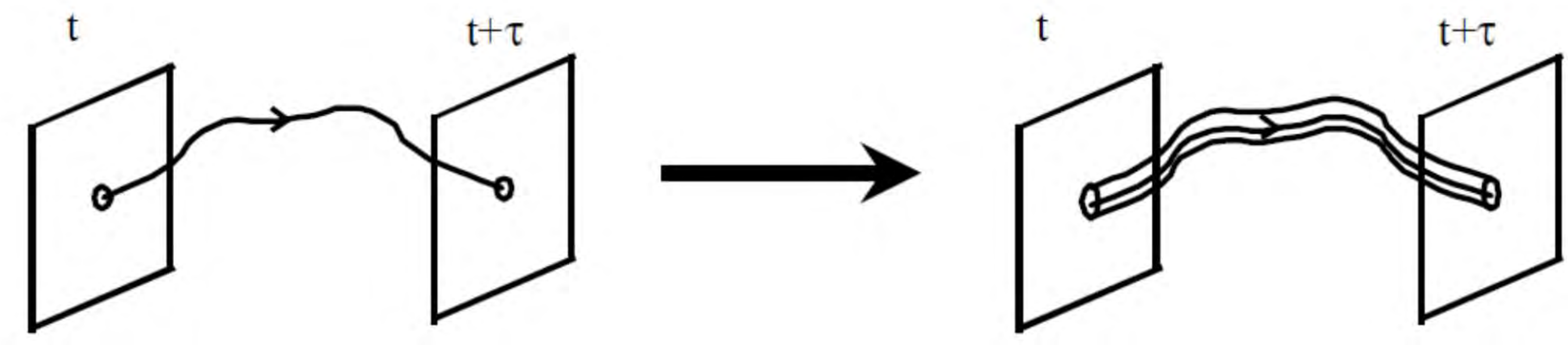
this perspective challenged by Poincaré and Sierpinski

Nonlinearity and Causality

Linear Systems

weak causality:
equal causes → equal effects

strong causality:
similar causes → similar effects

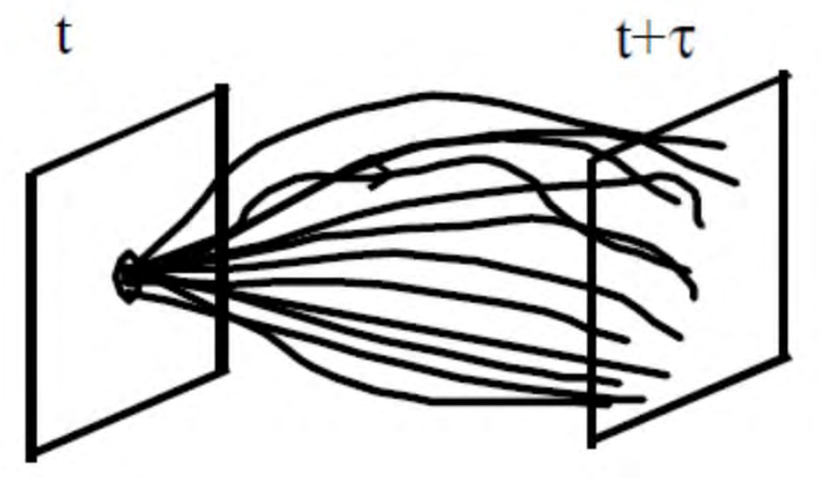


strong idealization;
does not account for
experimental conditions

includes weak causality;
accounts for experimental conditions:
tiny deviations from initial conditions,
weak perturbations, systematic errors, …

Nonlinearity and Causality **Nonlinear Systems**

violation of strong causality:
similar causes → vastly different effects



- sensitive dependence on initial conditions
- deterministic chaos
- pattern formation
- “the whole is more than the sum of its parts” (Aristoteles)
- self-organization

Processes and their Characteristics

regular process	chaotic process	stochastic process
deterministic	deterministic	stochastic (noise/randomness)
long-term predictable	predictable	non-predictable
strong causality	violation of strong causality	no causal relationships
	nonlinearity	

Deterministic Chaos

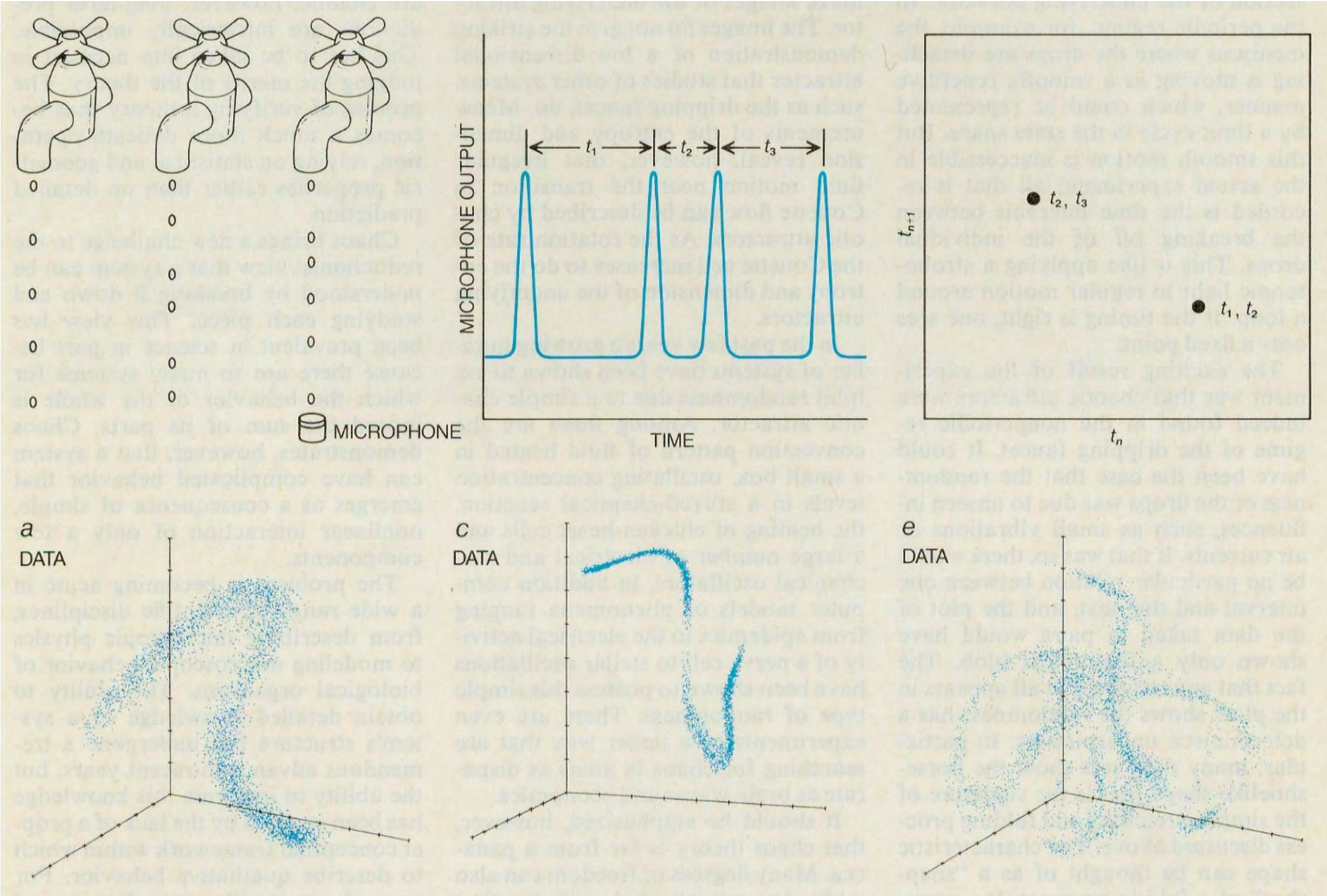
Chaos (colloquially)

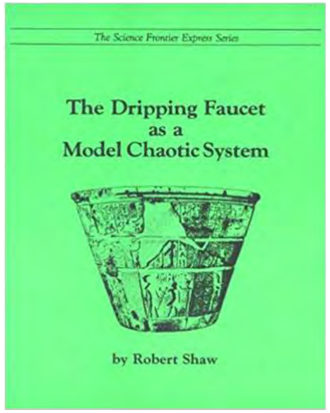
- disordered state and irregularity

Deterministic Chaos

- irregular (non-periodic) behavior
- non-predictable or for some short time horizon only
- deterministic equations of motion (in contrast to stochasticity)
- instabilities and recurrences

Deterministic Chaos dripping faucet





SCIENTIFIC AMERICAN 1986 254, 46-57.

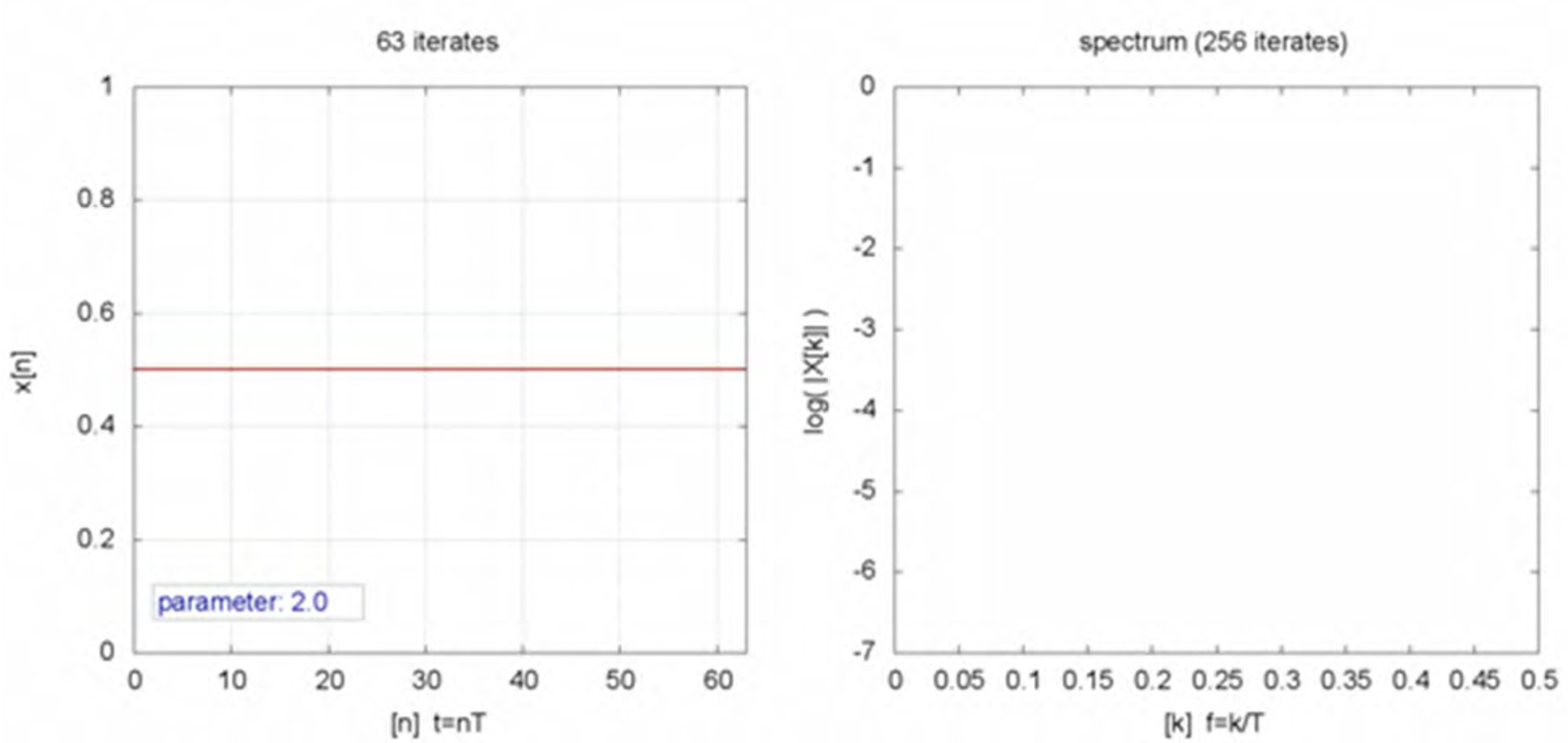
Deterministic Chaos **period doubling**

P.F. Verhulst

R. May

"chaotic behavior can arise from very simple non-linear dynamical equations": ***logistic map*** (model for population growth, 1837)

$$x_{n+1} = r x_n (1 - x_n); \quad x_n \in [0,1]; \quad r \in [0,4]$$



Deterministic Chaos period doubling

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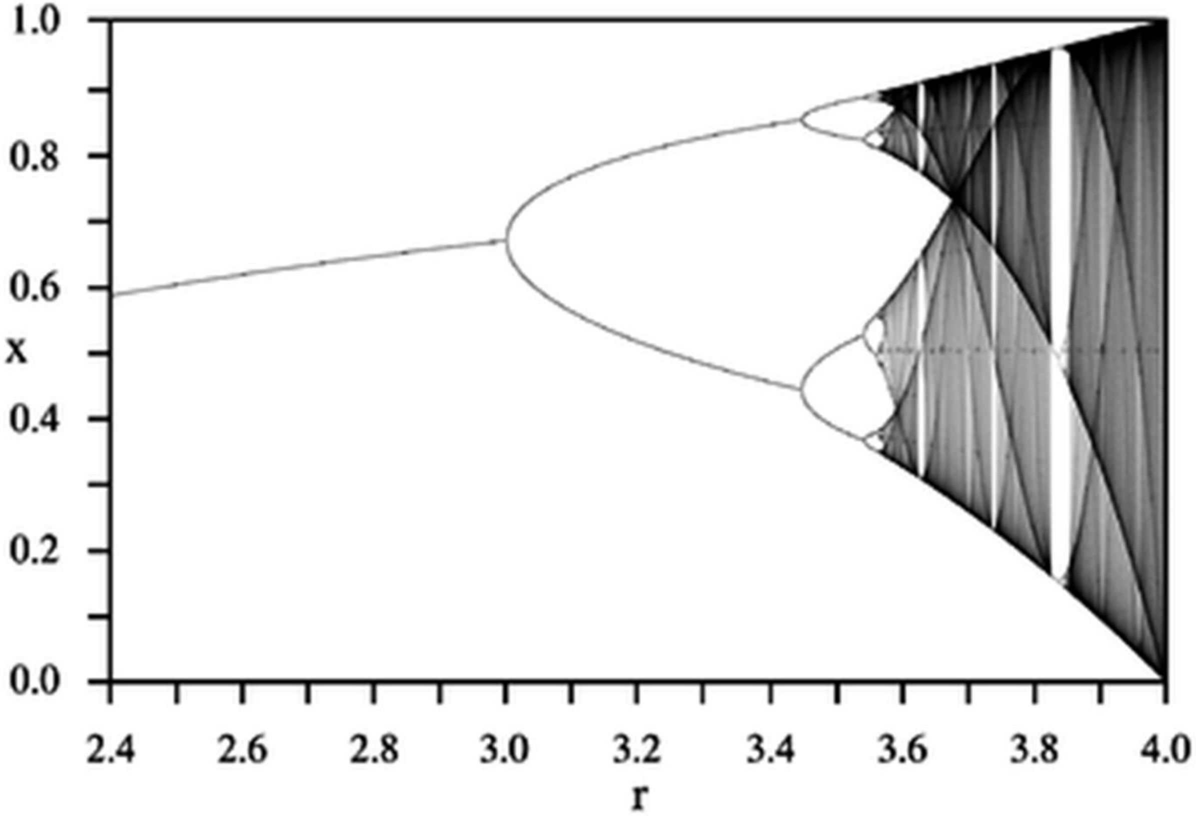
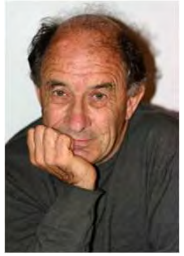


bifurcation diagram

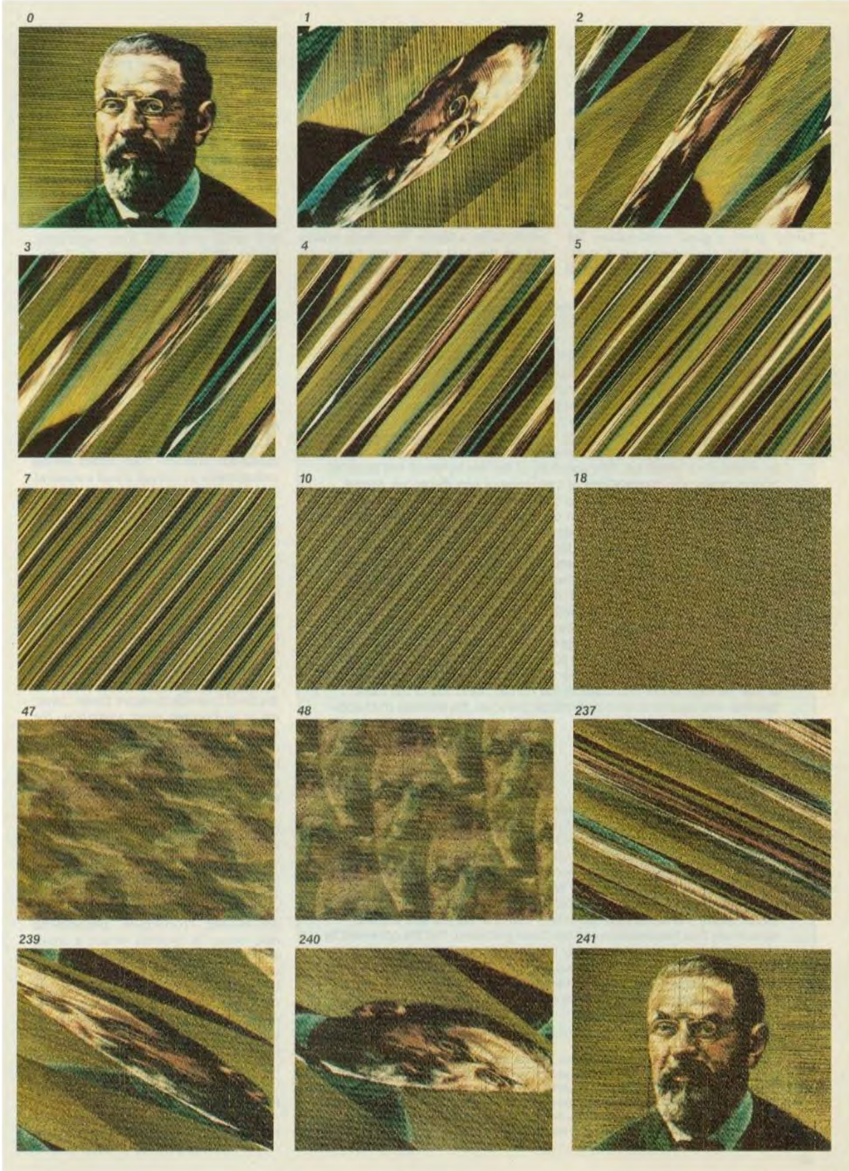
- period-doubling route to chaos
- period-3 implies chaos
- islands of stability
- periodicities within chaos
- self-similarity

Deterministic Chaos

recurrence and self-similarity



V. Arnold
(1937-2010)

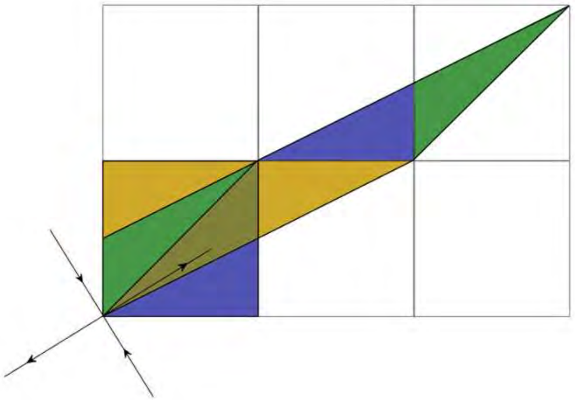


Arnold's cat map:

chaotic map from the torus into itself:

$$\Gamma : \mathbb{T}^2 \to \mathbb{T}^2$$

$$\Gamma : (x, y) \to (2x + y, x + y) \quad \mod 1$$



deterministic operations:
- stretching
- bending
- folding (nonlinear)

SCIENTIFIC AMERICAN 1986 254, 46-57.

Nonlinear dynamical systems

- can be described by nonlinear ODEs.
  However, no analytic solutions exist!

- show qualitatively rich dynamics:
  drastic changes upon changes of control parameters (bifurcation)
  deterministic chaos

- long-term behavior can be assessed by investigating the phase-space (state-space)