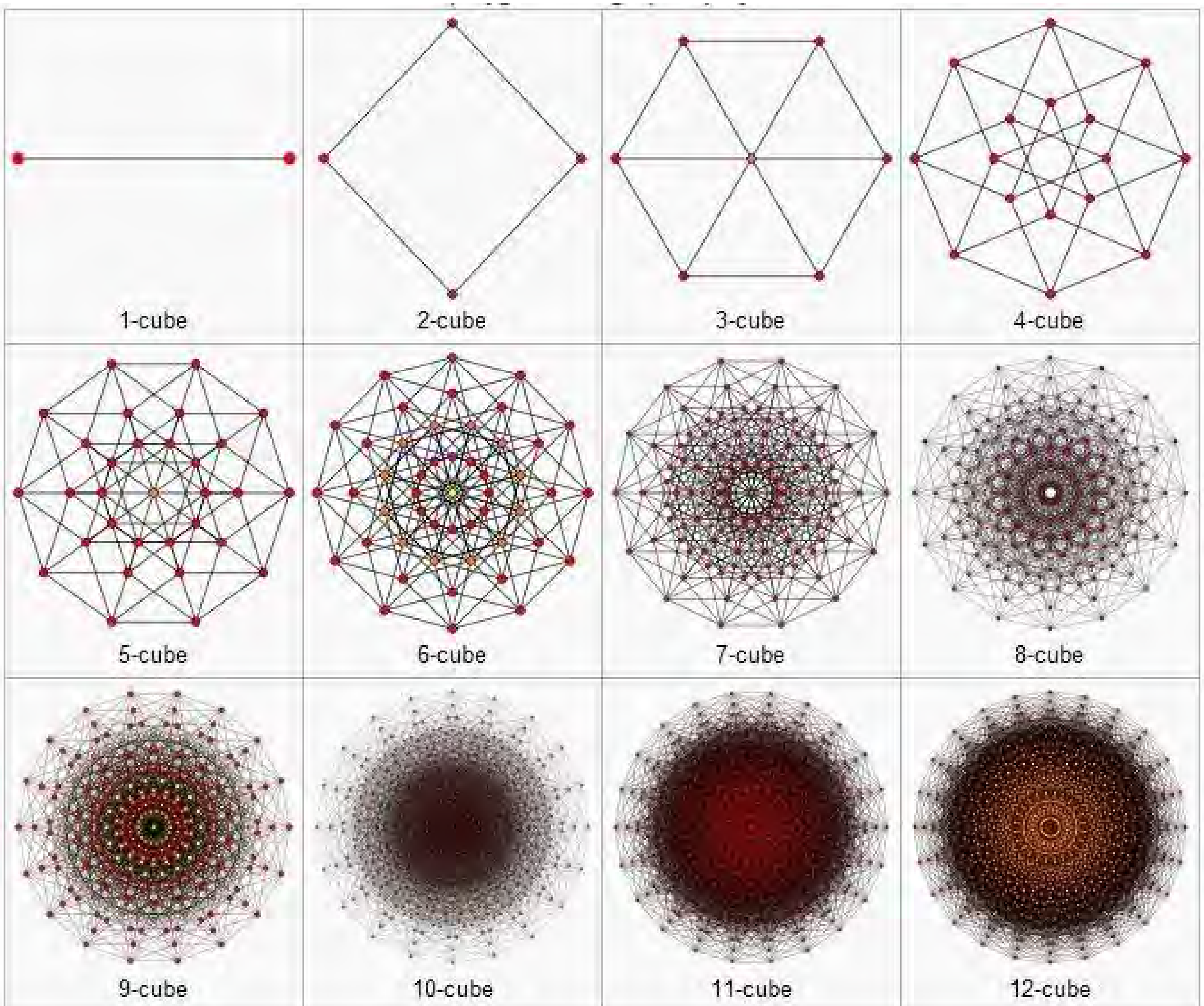
Dimensions, Fractals,

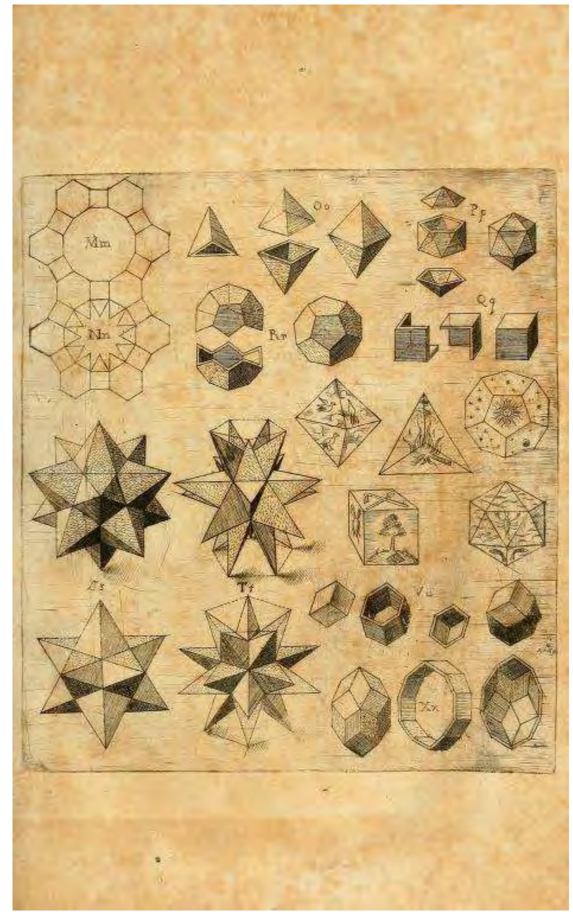
and

Dimensions from Time Series

Dimensions



from: autodeskresearch.com



Ioannis Keppleri: Harmonices mundi, 1619

Euclidean geometry

- characterization of some geometric object
- integer dimension

object	dimension
point	0
line	1
area	2
volume	3
n-cube	*n*

- number of degrees of freedom for characterization

Time series analysis: minimum number of equations needed to model a physical system, system complexity, number of degrees of freedom (see later)

Euclidean geometry and generalized dimension

Idea:

If you multiply all lengths by a,

- lengths will change by a factor a
- areas will change by a factor a^2
- volumes will change by a factor a^3
- …

→ determine dimension by exponent of content-scaling

Non-Euclidean geometry

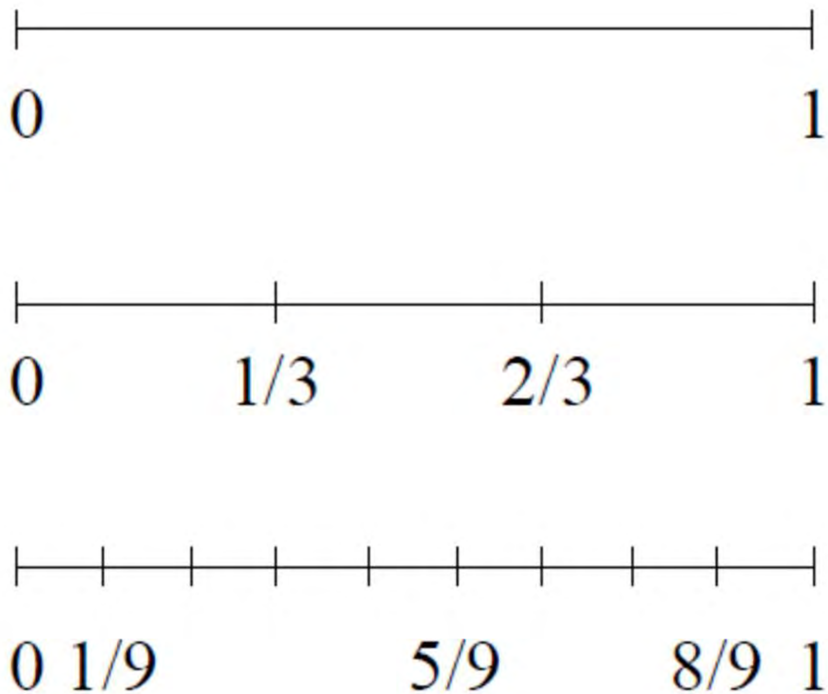
> generalized concept (F. Hausdorff, 1919)
> dimension of some non-Euclidean object in an *m*-dim. space
> idea:

- cover object in *m*-dim. space with hypercubes of side length ε
- determine minimum number $N(\varepsilon)$ of hypercubes necessary to fully cover object
- we find:

$$N(\epsilon) \underset{\epsilon \to 0}{\propto} \epsilon^{-D_0}$$

- Hausdorff dimension, fractal dimension, box-counting dimension ($D_0 = D_H$ in most cases)

Hausdorff dimension of a line



$\varepsilon = 1,\ N = 1$

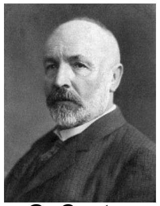
$\varepsilon = 1/3,\ N = 3$

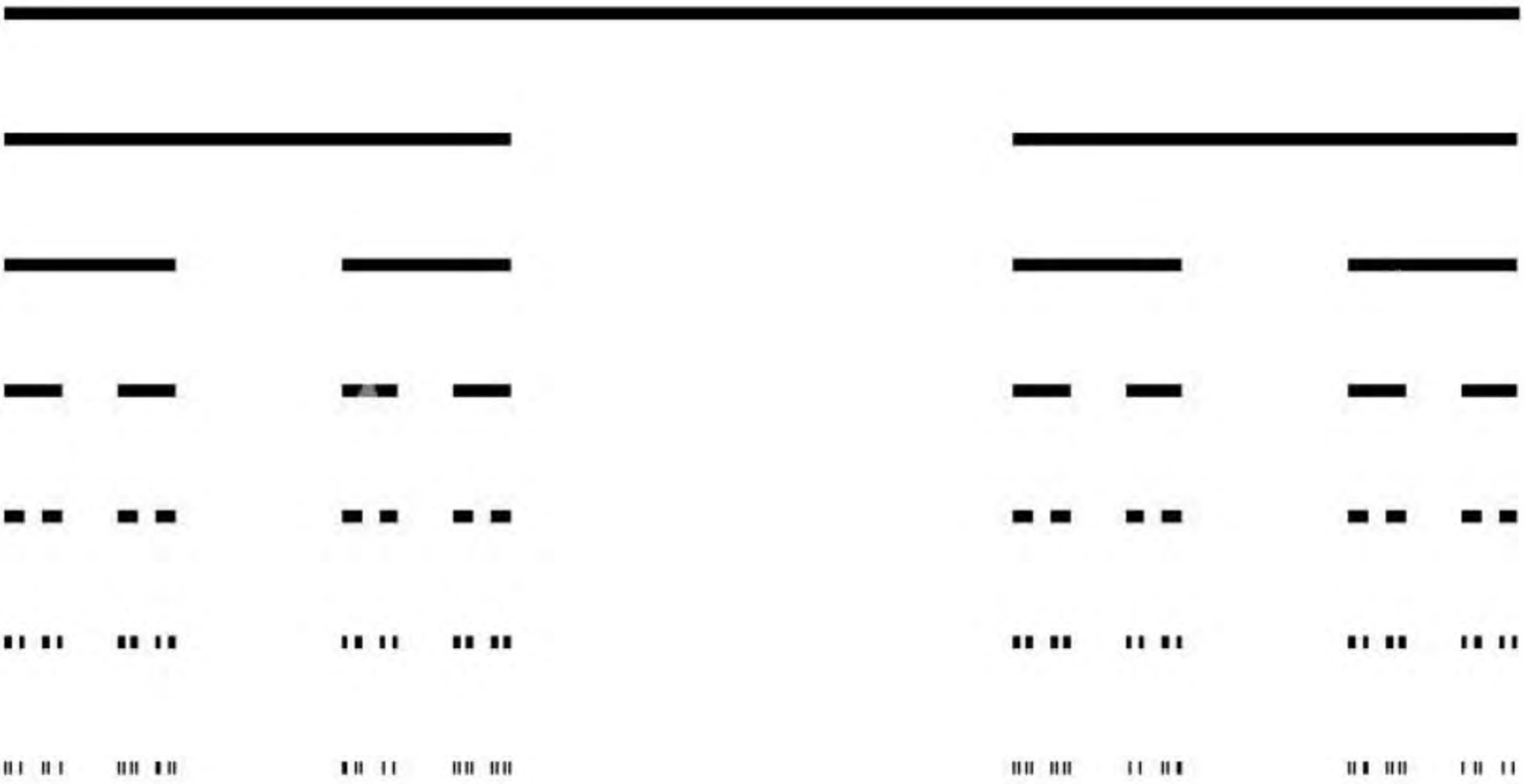
$\varepsilon = 1/9,\ N = 9$

$$D_0 = \frac{\log\left[N(\varepsilon)\,/\,N(\varepsilon')\right]}{\log(\varepsilon\,/\,\varepsilon')} = \frac{\log 3}{\log 3} = 1$$

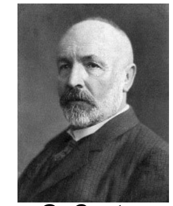
Hausdorff dimension of Cantor set


H.J.S. Smith

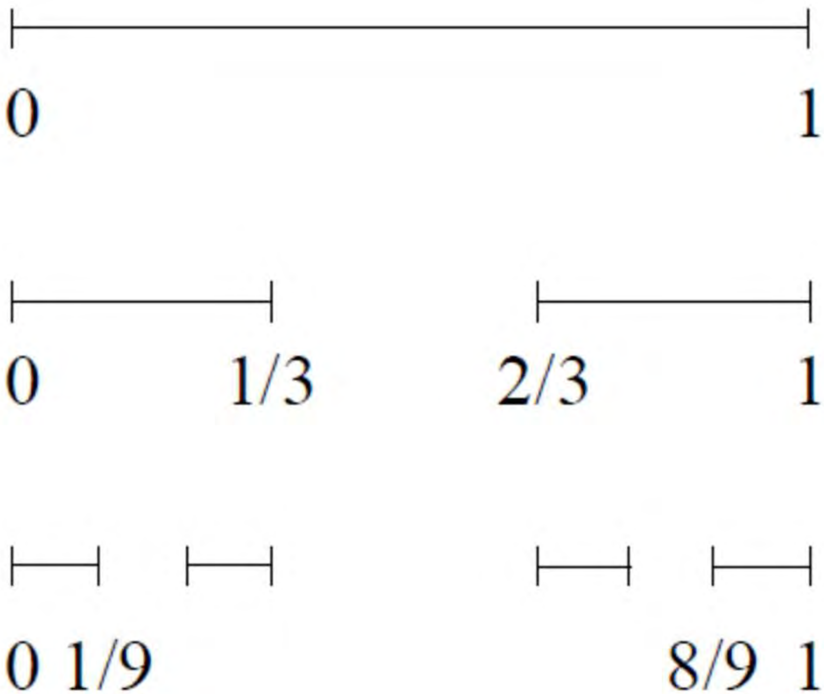

G. Cantor




H.J.S. Smith


G. Cantor

Hausdorff dimension of Cantor set



$\varepsilon = 1,\ N = 1$

$\varepsilon = 1/3,\ N = 2$

$\varepsilon = 1/9,\ N = 4$

$$D_0 = \frac{\log[N(\varepsilon)\,/\,N(\varepsilon')]}{\log(\varepsilon\,/\,\varepsilon')} = \frac{\log 2}{\log 3} = 0{,}6309$$

Fractals


Benoît B. Mandelbrot

Def.: a set $\mathcal{F}$ is called a ***fractal***, if

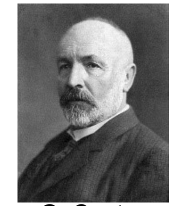
- $\mathcal{F}$ has some fine structure
- $\mathcal{F}$ *is* irregular
- $\mathcal{F}$ shows self-similarity (a subset of $\mathcal{F}$ is similar to $\mathcal{F}$)
- fractal (Hausdorff-Besicovitch) dimension strictly exceeds the topological dimension

Applications:

many natural structures, modelling, technology, art, …

Hausdorff and topological dimension of Cantor set


H.J.S. Smith


G. Cantor

$$D_0 = \frac{\log\left[N(\varepsilon) / N(\varepsilon')\right]}{\log(\varepsilon / \varepsilon')} = \frac{\log 2}{\log 3} = 0{,}6309$$

Length *L* (topological dimension)

$$L = 1 - \frac{1}{3} - \frac{2}{9} - \frac{4}{27} \ldots = 1 - \frac{1}{3}\sum_{\upsilon=0}^{\infty}\left(\frac{2}{3}\right)^{\upsilon} = 0$$

$$D_0 > L$$

Fractals

Benoît B. Mandelbrot

Q: What does the “B.” in “Benoît B. Mandelbrot” stand for?

A: “Benoît B. Mandelbrot”

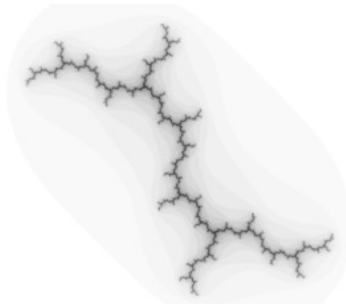
possibly not a joke

Fractals and the Coastline Paradox

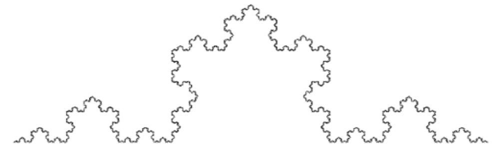
If the coastline of Great Britain is measured with a ruler of 100 km length, then the length of the coastline is about 2800 km.
With a ruler of 50 km length, the total length is about 3400 km, i.e. 600 km longer.
For rulers with smaller length, the length of the coastline diverges to infinity

Some Fractals and their dimensions

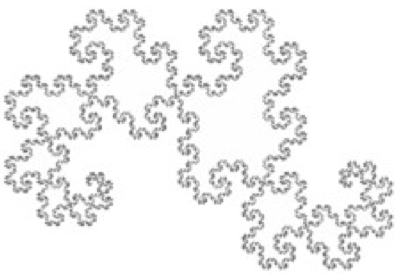
Cantor set
($\log_3(2)=0.6309$)



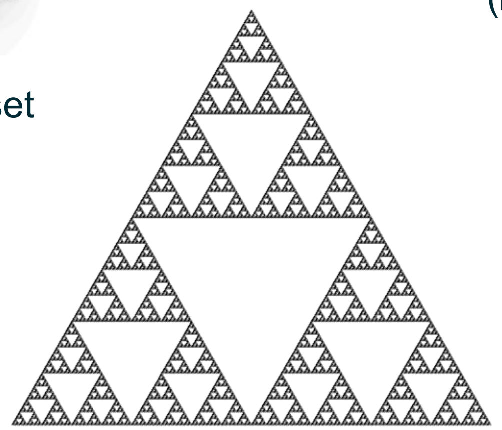
dendrite of Julia set
(1.2)



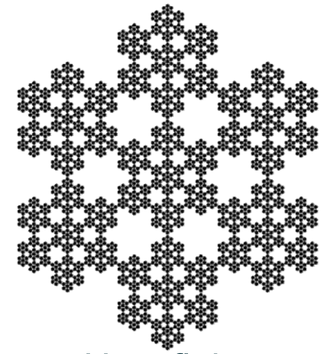
Koch curve
($\log_3(4)=1.2619$)



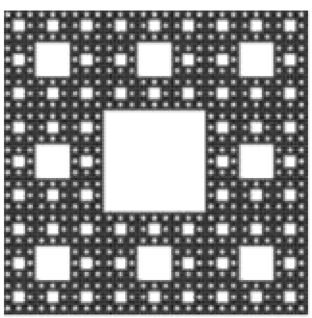
boundary of dragon curve
(1.5236)



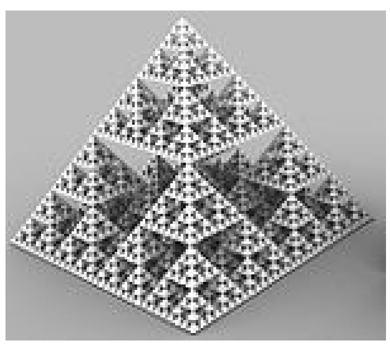
Sierpinski triangle
(1.5849)



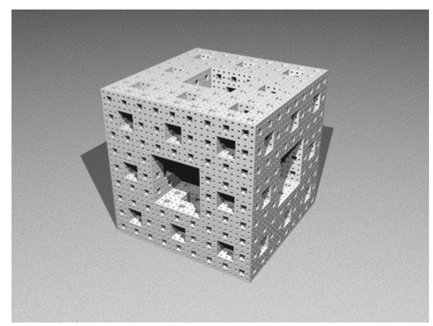
Hexaflake
($\log_3(7)=1.7712$)



Sierpinski carpet
($\log_3(8)=1.8928$)



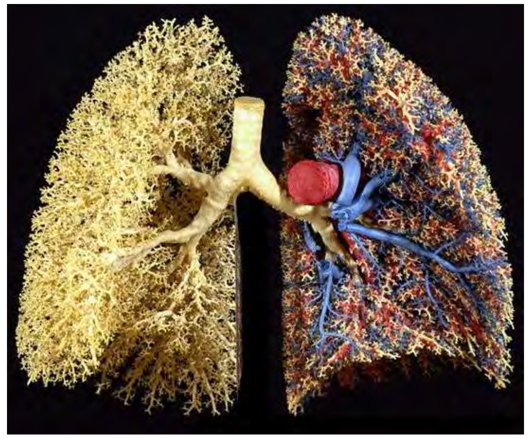
fractal pyramid
($\log_2(5)=2.3219$)



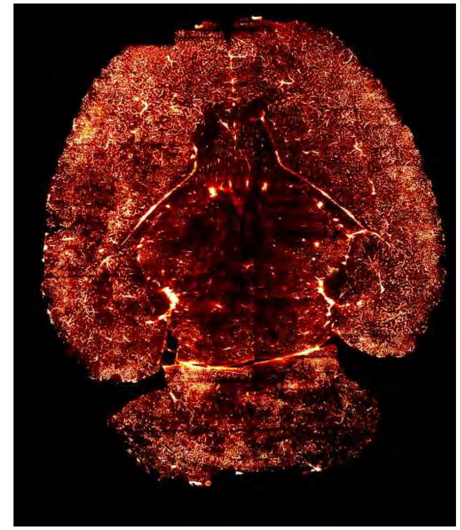
Menger sponge
($\log_3(20)=2.7268$)

Some Natural Fractals

Romanesco broccoli



lung



blood vessels mouse brain

fern

river delta

dandelion

bolt

Some Natural Fractals

>> Physics News Update, 92, 19 August 1992

The landscape of DNA may be fractal

(Phys Rev Lett 22 Jun 92)

>> Physics News Update, 353, 5 Januar 1998

Fractal patterns inside cells can reveal breast cancer

(Phys Rev Lett, 12 Jan 98)

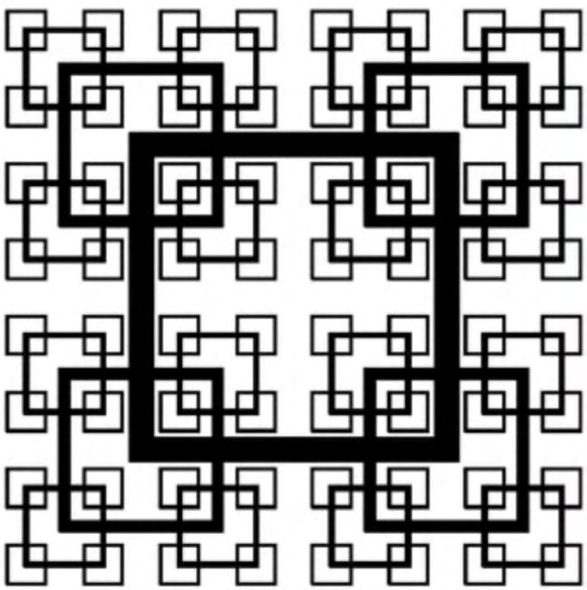
>> Physics News Update, 399, 26 Oktober 1998

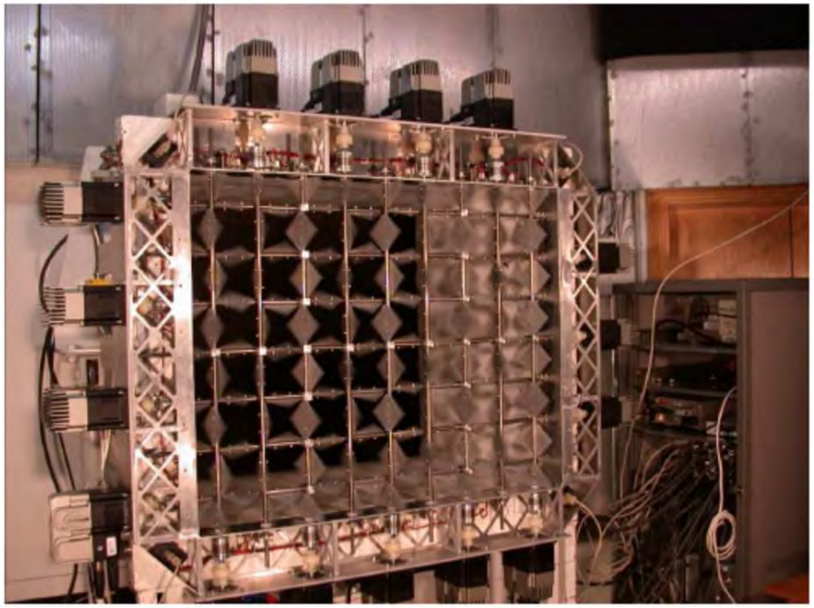
Tumor growth can be fractal (fractal dimension: 1.21)

(Phys Rev Lett, 2 Nov 98)

Technical Fractals (an example)

fractal grids to generate turbulent flow

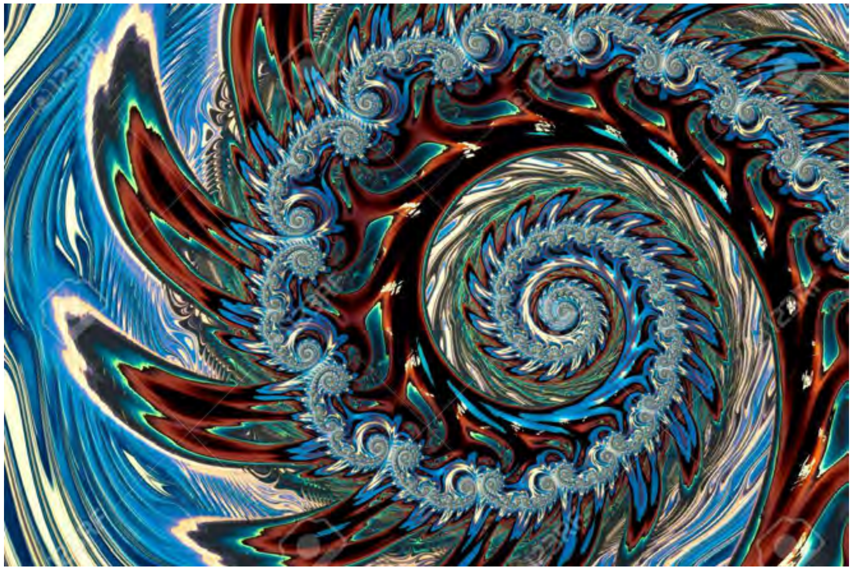


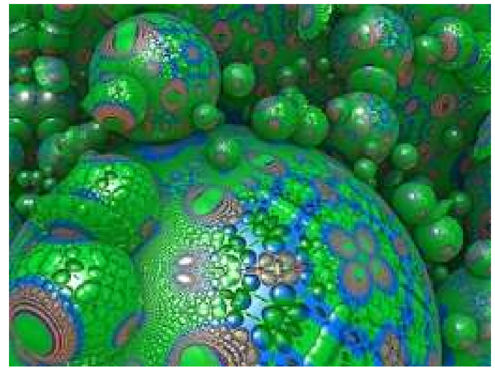


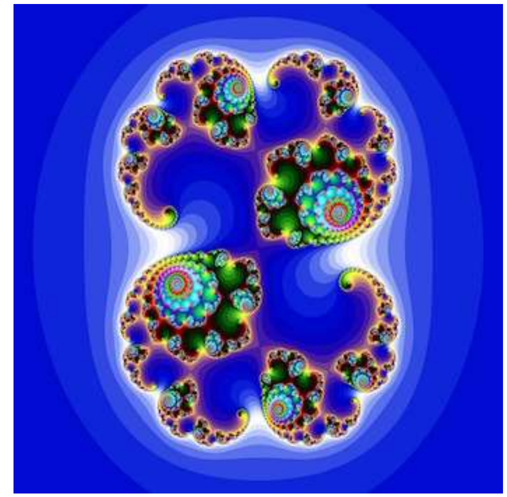
towards "fractal wind turbine blades"

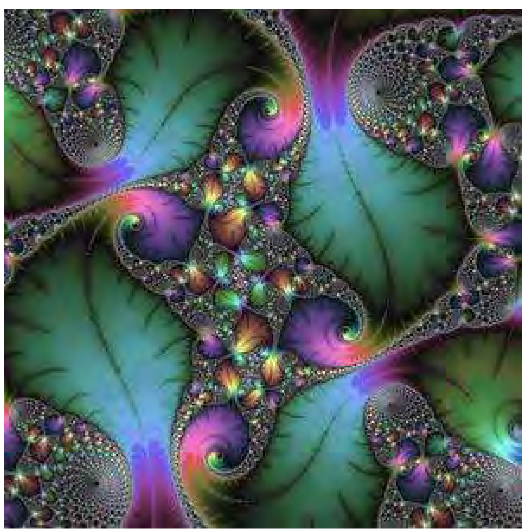
S. Weitemeyer et al. Multi-scale generation of turbulence with fractal grids and an active grid. Fluid Dynamics Research 45(6):061407, 2013

Fractals and Art









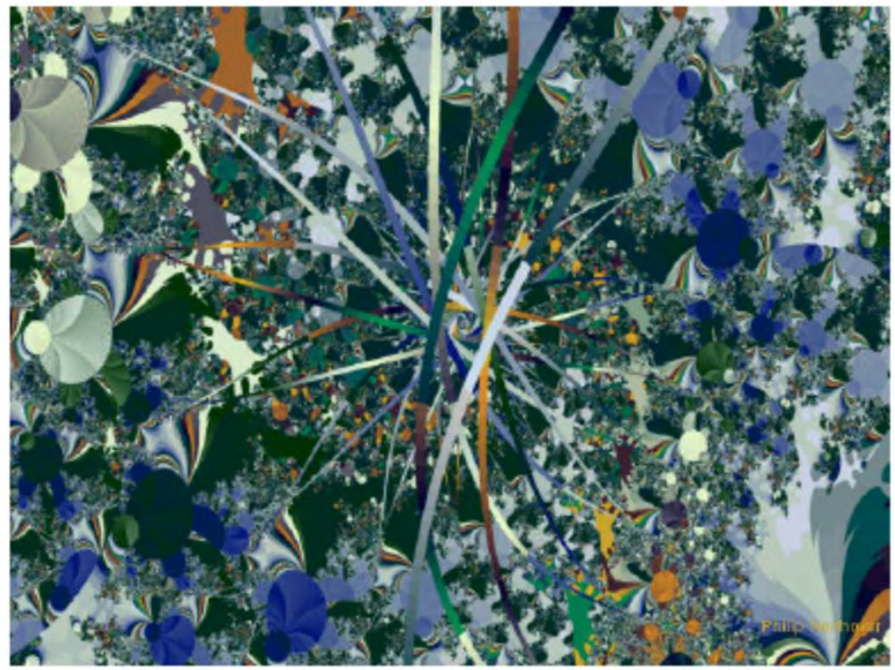
Fractals and Art

Jack the dripper: chaos in modern art

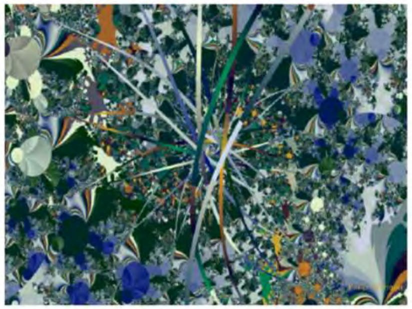
Of all the abstract expressionist painters, Jackson Pollock was perhaps the most controversial. He would dash around large canvases rolled out on the floor of his barn, dripping paint from a wooden stick. The critics poured scorn on his paintings, calling them "meaningless chaos". But chaos is now a rigorous scientific concept that we know appears throughout nature. One important part of chaos theory is fractal behaviour, which describes objects that have similar patterns when viewed at different magnifications. Richard Taylor, a physicist at the University of New South Wales, has now discovered this characteristic in many of Pollock's works. Rather than being the fraud that many people assume, Taylor believes that Pollock subconsciously understood the patterns of nature so well that he was able to capture their very essence -- chaos and fractals -- on canvas.

Physics Web November 1997

Fractals and Art



Fractals and Art



Fractals determine date of paintings

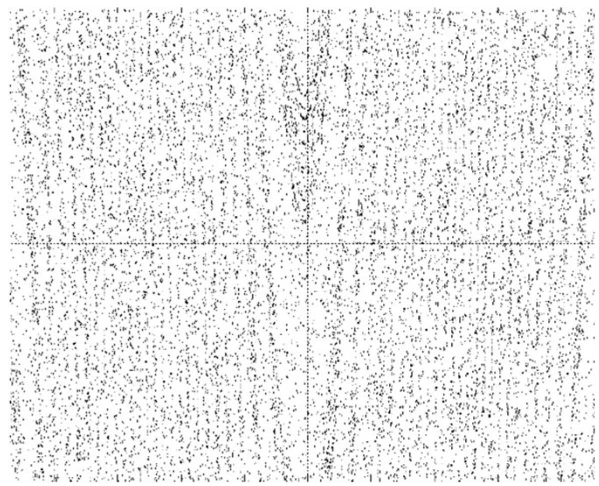
[4 Jun 1999] Paintings by the late Jackson Pollock - considered to be one of the fathers of modern art - can be dated by fractal geometry according to Australian physicists (Nature 399 422). Pollock's artwork during the late 1940s consisted of paint dripped from a can onto large canvases spread out on the floor of his barn. Richard Taylor, Adam Micolich and David Jonas from the University of New South Wales in Sydney discovered that ***the fractal dimension of Pollock's drip paintings increased from nearly 1.0 in 1943, to 1.72 in 1952***, suggesting that Pollock gradually refined his technique over to time to make his painting more fine grained.

Physics News 1999

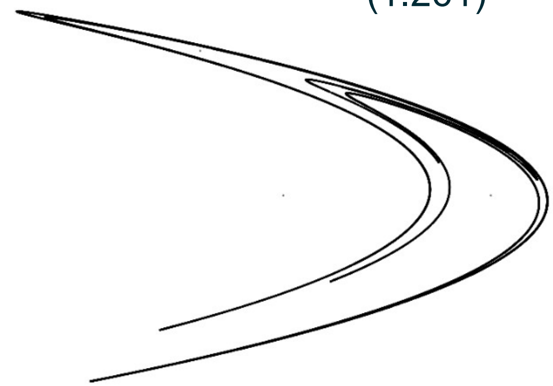
Strange attractors and fractal dimensions

Ikeda map
(1.7)

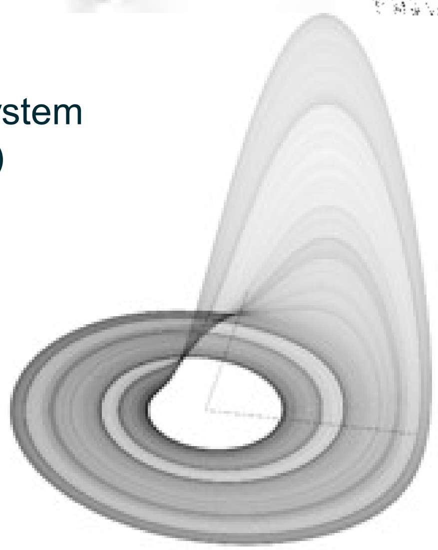
Zaslavskii map
(1.39 ?)



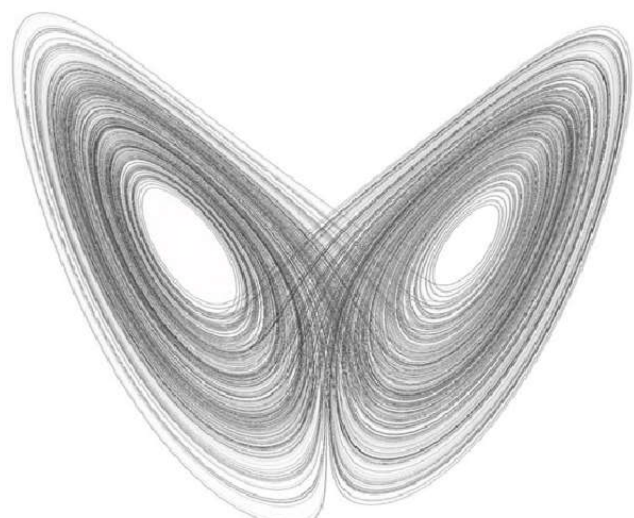
Hènon map
(1.261)



Rössler system
(2.01)



Lorenz system
(2.06)



Generalized dimensions

- estimating dimensions in high-dimensional space via box-counting is hard
- box counting ignores how densely the boxes are populated
- idea: weight boxes by probability p_i to find state in box i

Rényi dimensions, q-dimensions

- partition state space into M hypercubes (boxes) of side length ε
- estimate probability by $p_i = \lim_{N \to \infty} {}^{N_i}/_{N}$

$$D_q := \lim_{\epsilon \to 0} \frac{\log\left(\sum_{i=1}^{M(\epsilon)} p_i^q\right)}{(q-1)\log(\epsilon)}$$

Generalized dimensions Rényi dimensions

- $D_0 := \lim_{q \to 0} D_q$ (box-counting dimension)
- $D_1 := \lim_{q \to 1} D_q$ (information dimension)
- D_2 (correlation dimension)
- $D_{topo} \leq D_H \leq \ldots \leq D_2 \leq D_1 \leq D_0 \leq m$
- in most cases: $D_H = \ldots = D_2 = D_1 = D_0$

D_0 counts non-empty boxes
D_1 measures gain of information to find state in box i
if $D_0 = D_1$, attractor is homogeneous

Generalized dimensions Rényi dimensions

dimension of attractors:

- $D \notin \mathbb{N}$: strong indicator for nonlinearity (chaotic dynamics)
  (D diverges for purely stochastic dynamics)
- characterizes self-similarity, complexity
- provides hints for modelling (degrees of freedom, attractor structure)
- sanity check via embedding theorems

Generalized dimensions Rényi dimensions

dimension from time series:

we have:

$$D_q := \lim_{\epsilon \to 0} \frac{\log\left(\sum_{i=1}^{M(\epsilon)} p_i^q\right)}{(q-1)\log(\epsilon)}$$

- replace limes by slope in double-logarithmic plot
- approximate $p_i \approx {}^{N_i}/_{N}$
- approximate sum over probabilities by “correlation sum”:

$$C_q(\epsilon) := \frac{1}{N}\sum_i \left(\underbrace{\frac{1}{N}\sum_j \Theta\left(\epsilon - |\vec{v_i} - \vec{v_j}|\right)}_{\text{counts number of point closer than } \varepsilon}\right)^{q-1}$$

counts number of point closer than ε

P. Grassberger and I. Procaccia, Physica D **9**, 198 (1983); Phys. Rev. Lett. **50**, 346 (1983).

Generalized dimensions Rényi dimensions

dimension from time series:

correlation dimension:

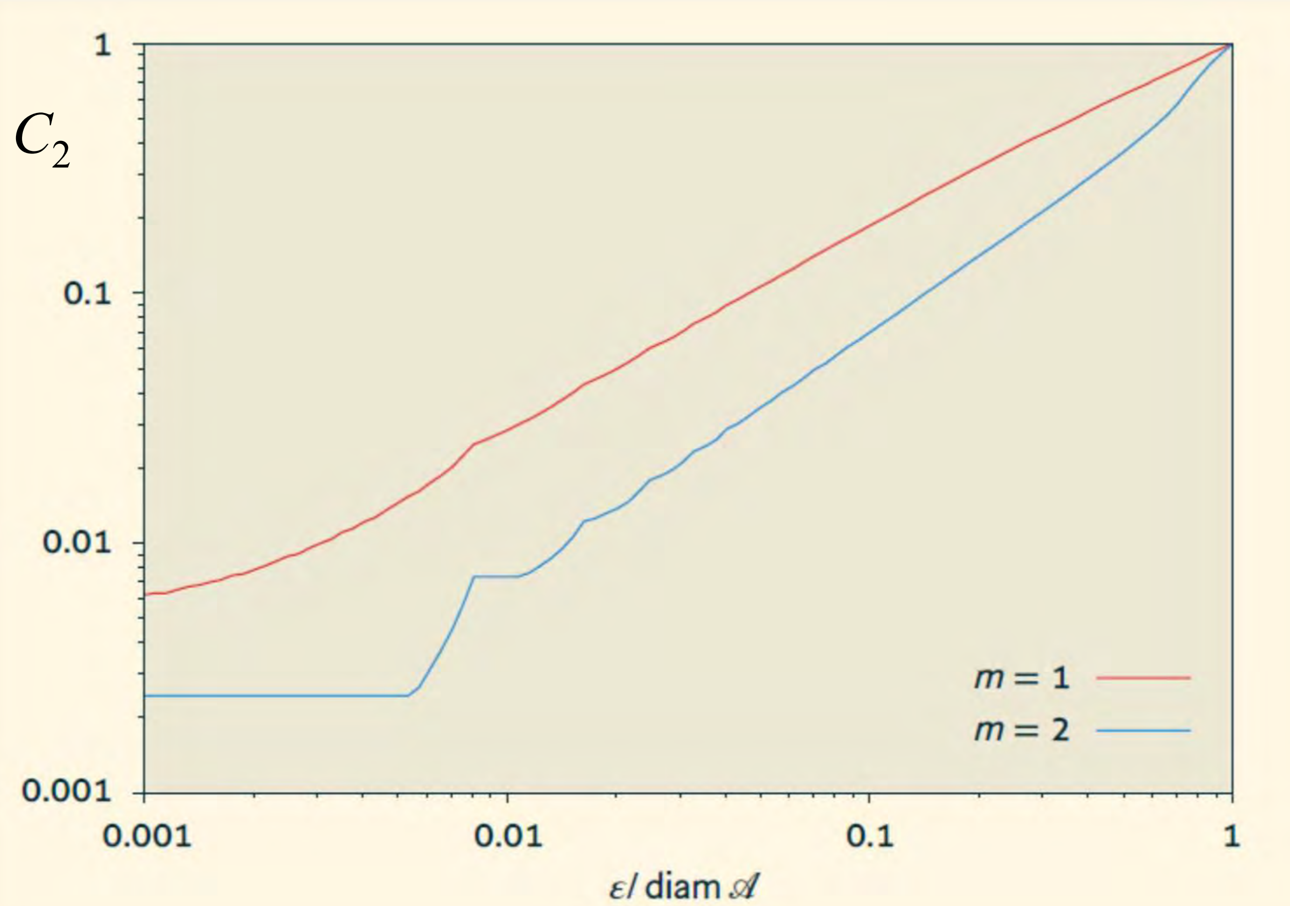
$$D_2 := \lim_{\epsilon \to 0} \frac{\log\left(\sum_{i=1}^{M(\epsilon)} p_i^2\right)}{\log(\epsilon)} \approx \lim_{\epsilon \to 0} \frac{\log(C_2(\epsilon))}{\log(\epsilon)}$$

- quickest to calculate

P. Grassberger and I. Procaccia, Physica D **9**, 198 (1983); Phys. Rev. Lett. **50**, 346 (1983).

Generalized dimensions **Rényi dimensions**

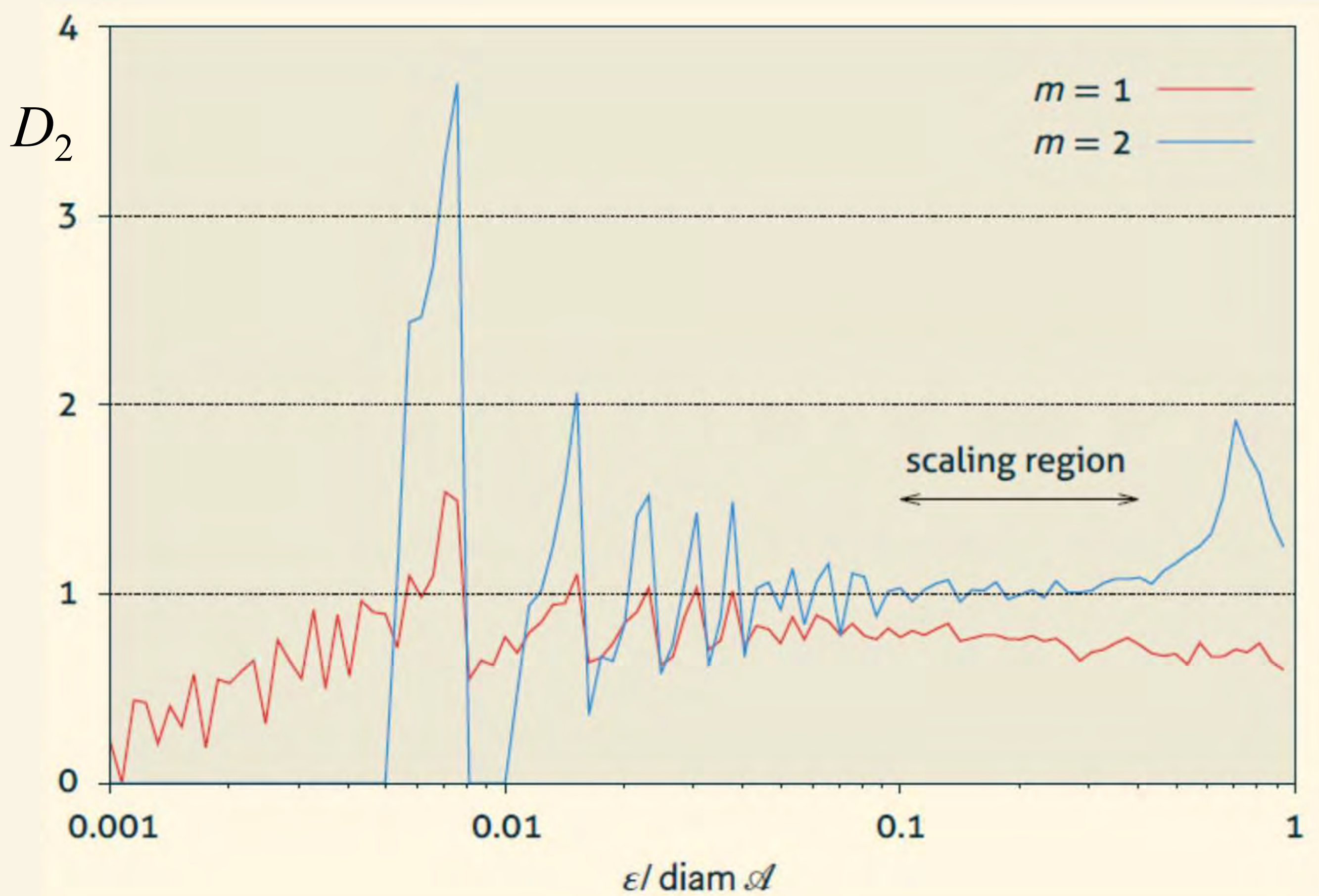
dimension from time series: **example: sine wave**



from: G. Ansmann

Generalized dimensions — Rényi dimensions

dimension from time series: example: sine wave



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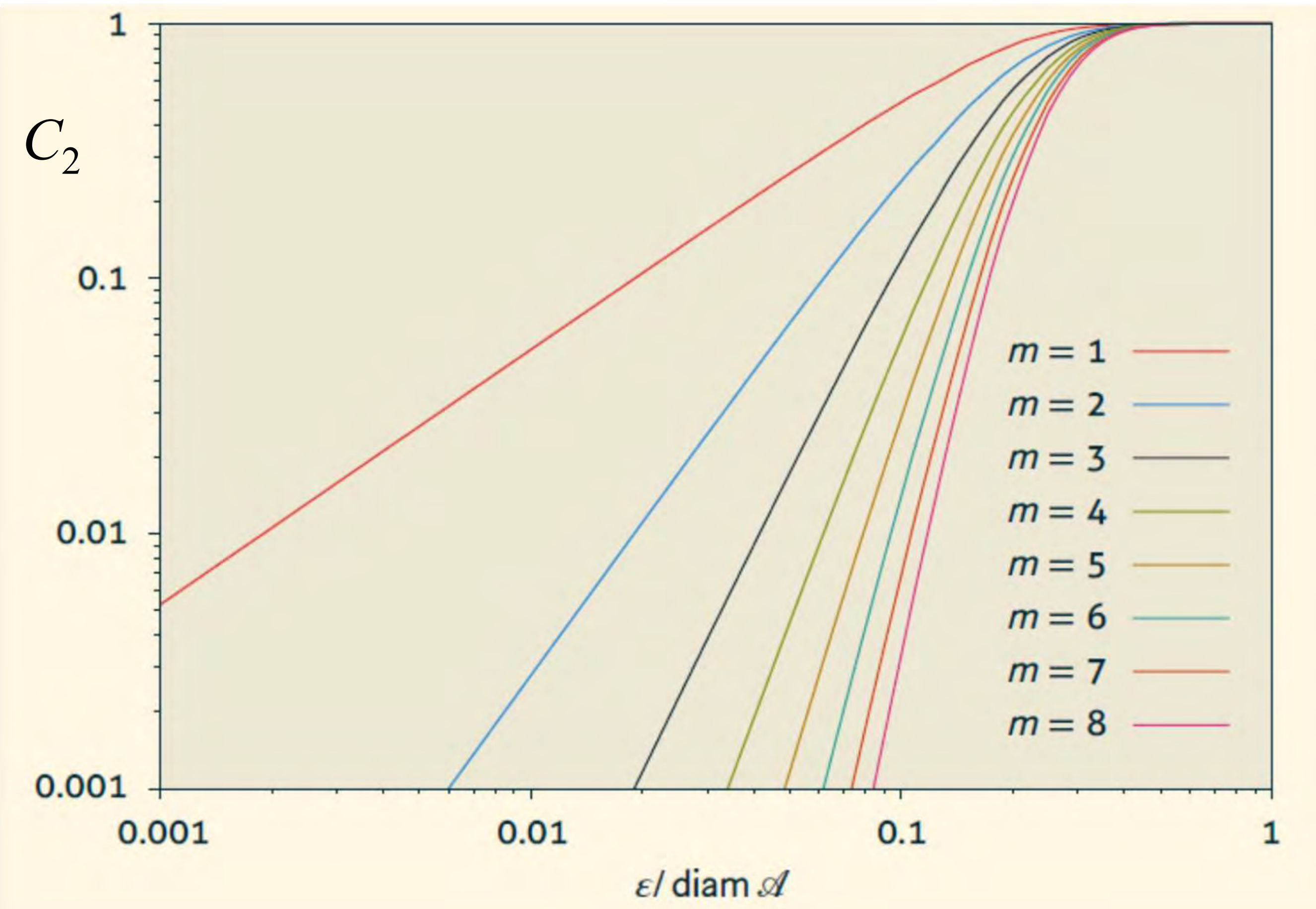
Generalized dimensions **Rényi dimensions**

dimension from time series: **example: Gaussian noise**

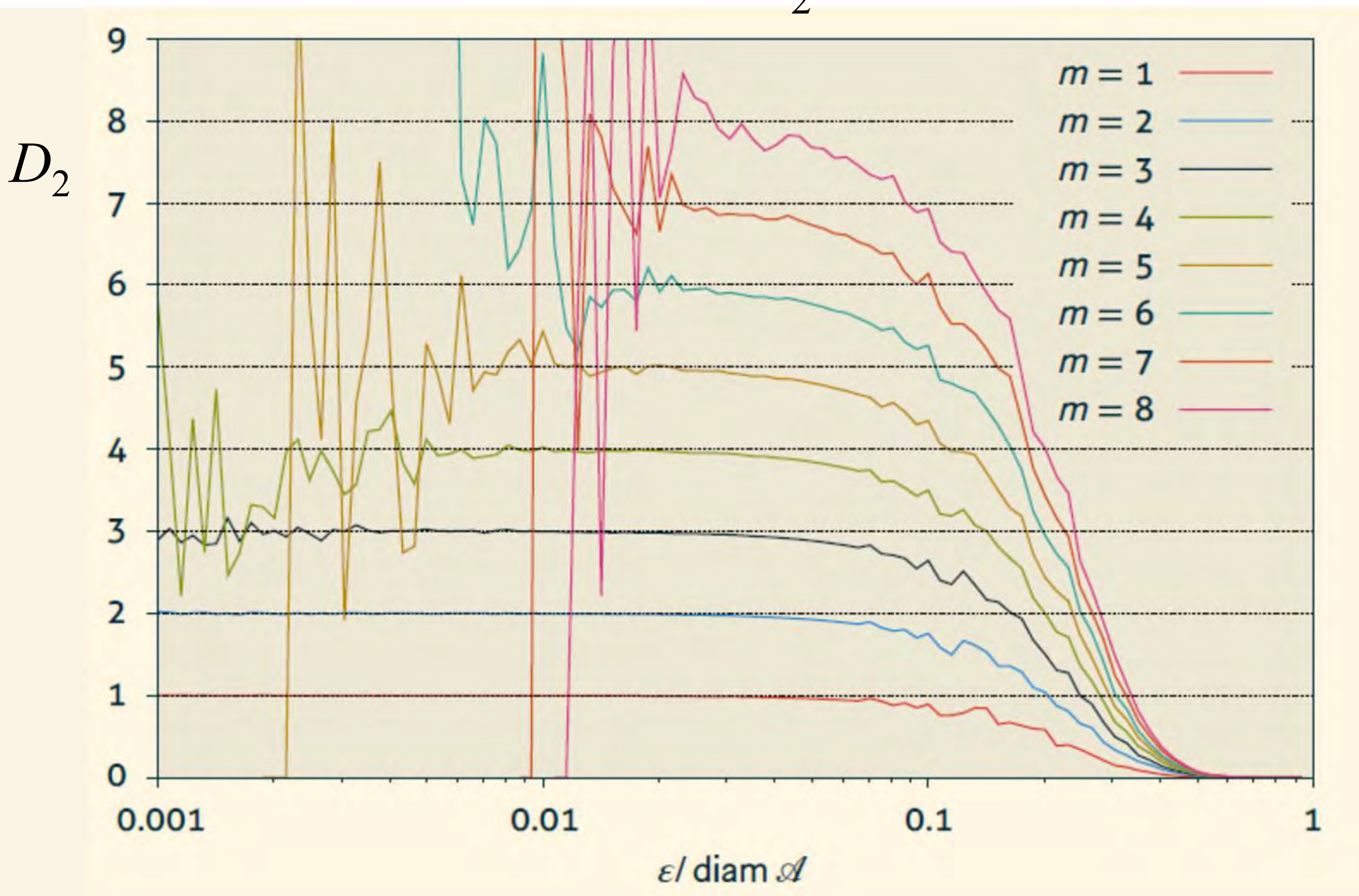


from: G. Ansmann

Generalized dimensions Rényi dimensions

dimension from time series: example: Gaussian noise

$D_2 \sim m$



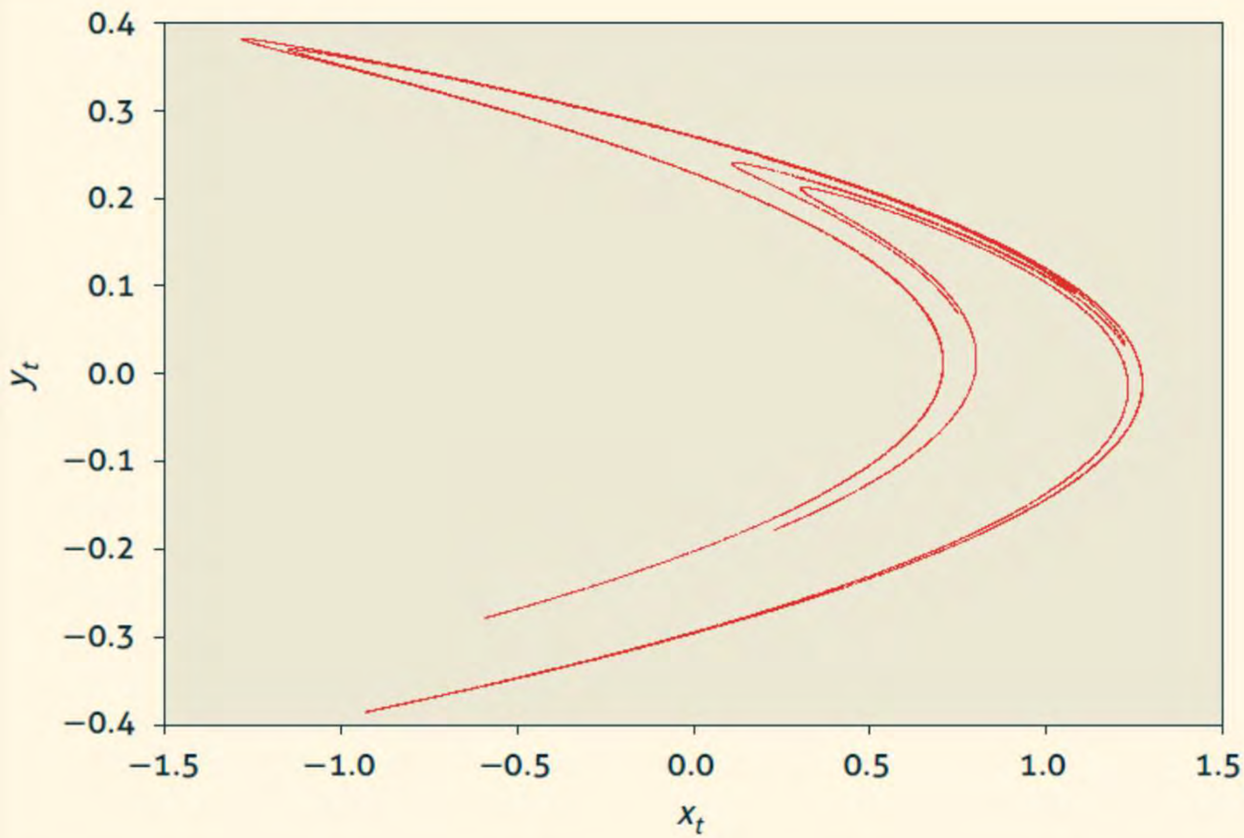
from: G. Ansmann

Generalized dimensions **Rényi dimensions**

dimension from time series: **example: Hénon map**

$$x_{n+1} = 1 - a x_n^2 + y_n$$
$$y_{n+1} = b x_n$$

where
$a = 1.4; b = 0.3$



from: G. Ansmann

Generalized dimensions — Rényi dimensions

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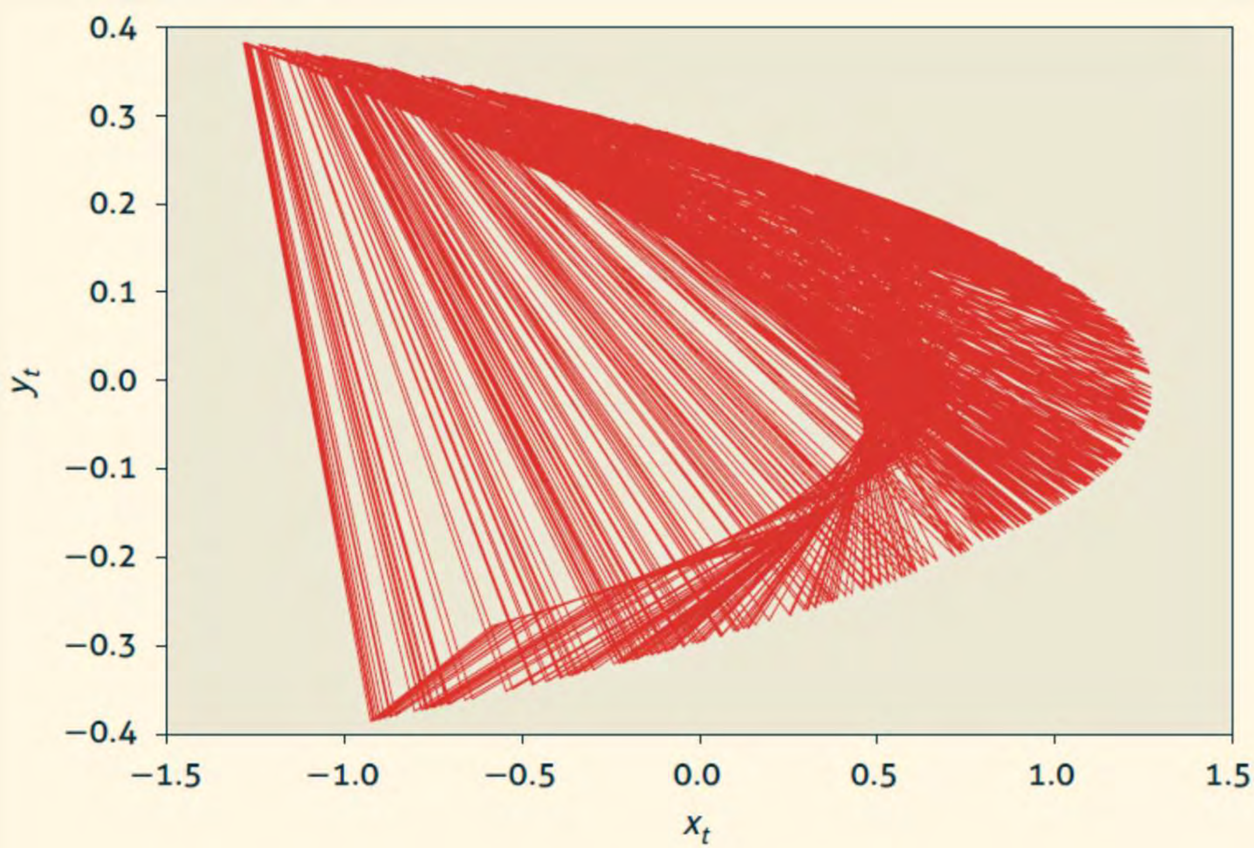

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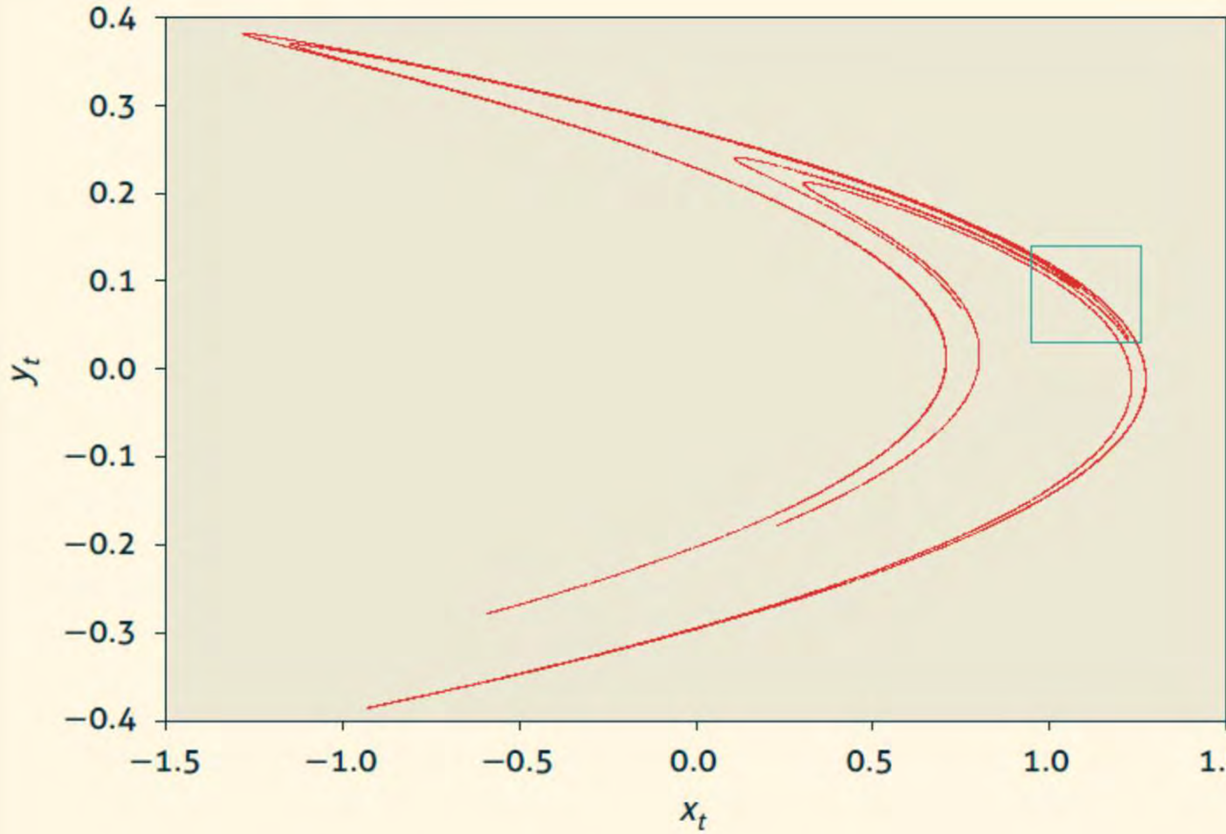

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Generalized dimensions — Rényi dimensions

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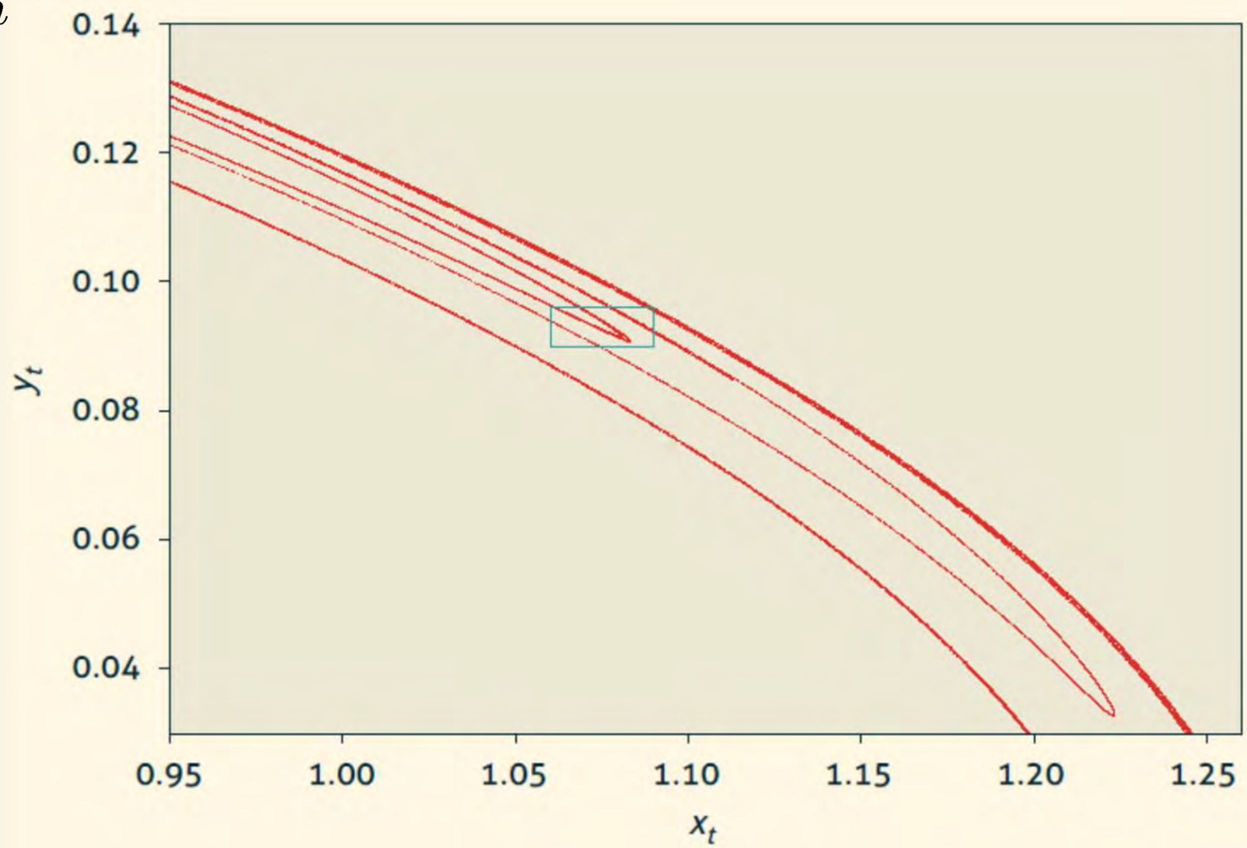

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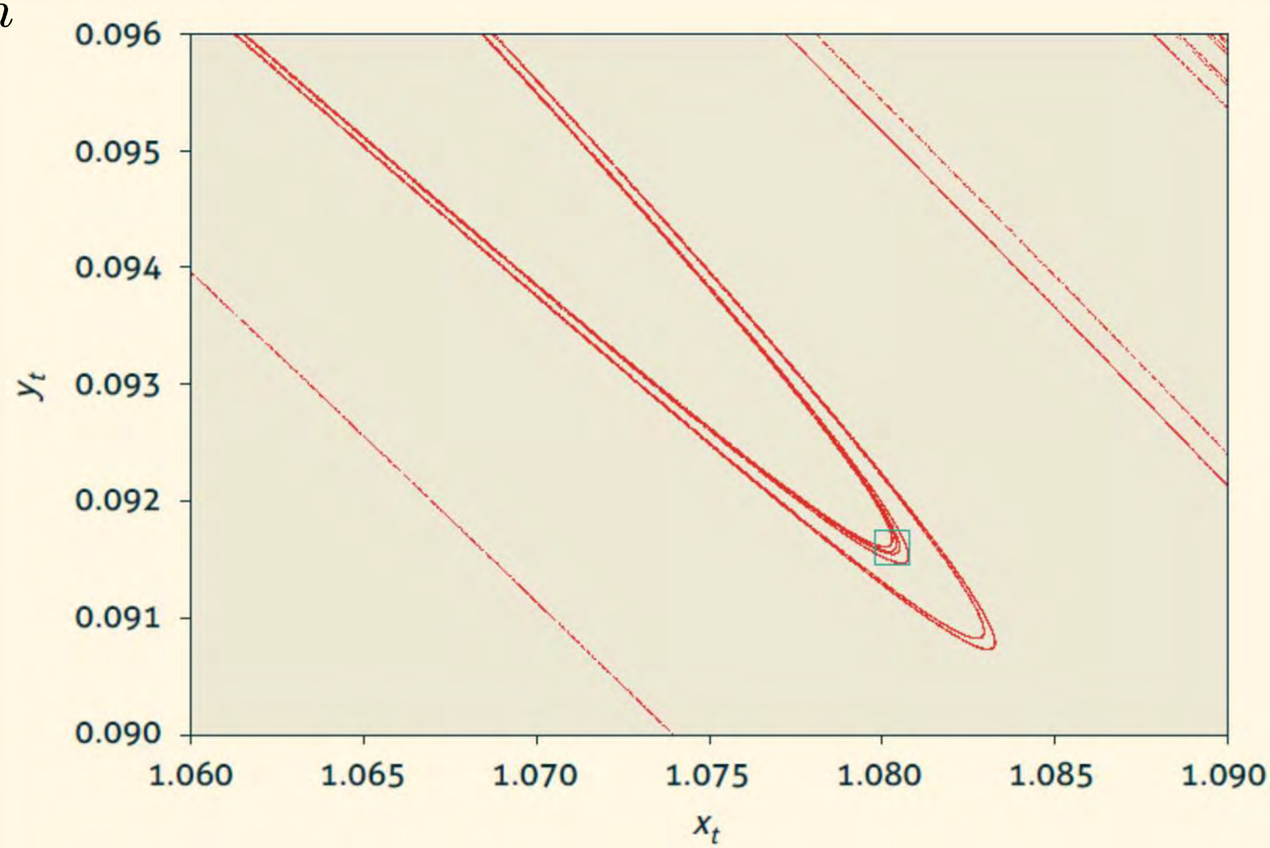

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Generalized dimensions — Rényi dimensions

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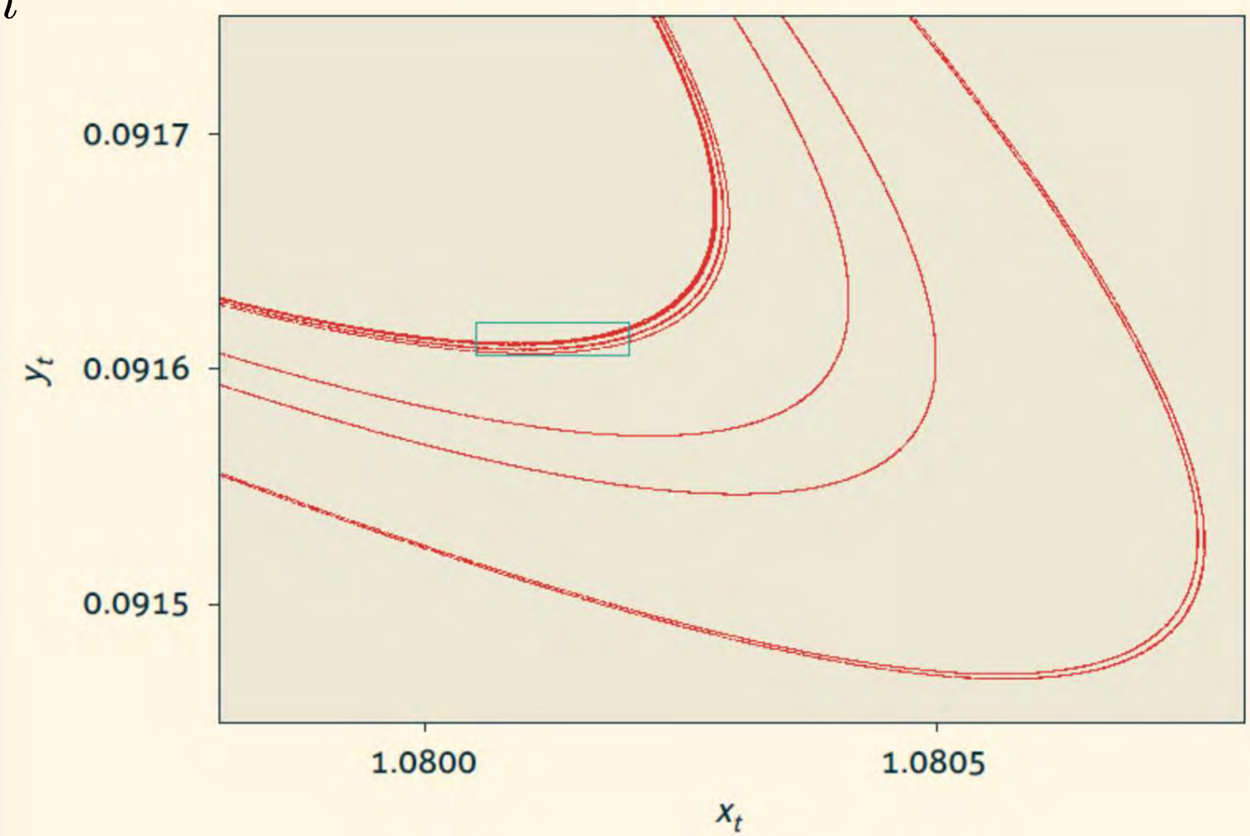

from: G. Ansmann

Generalized dimensions — Rényi dimensions

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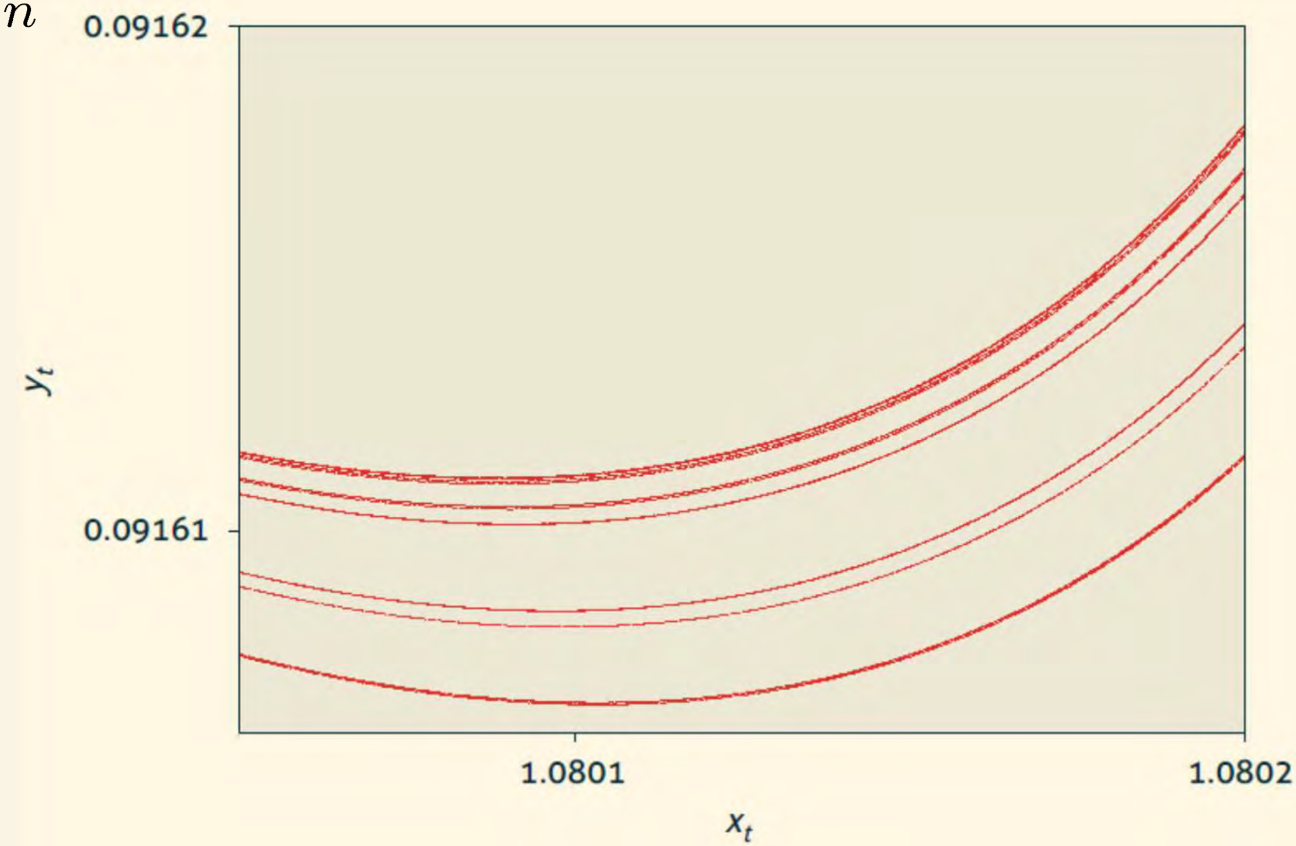

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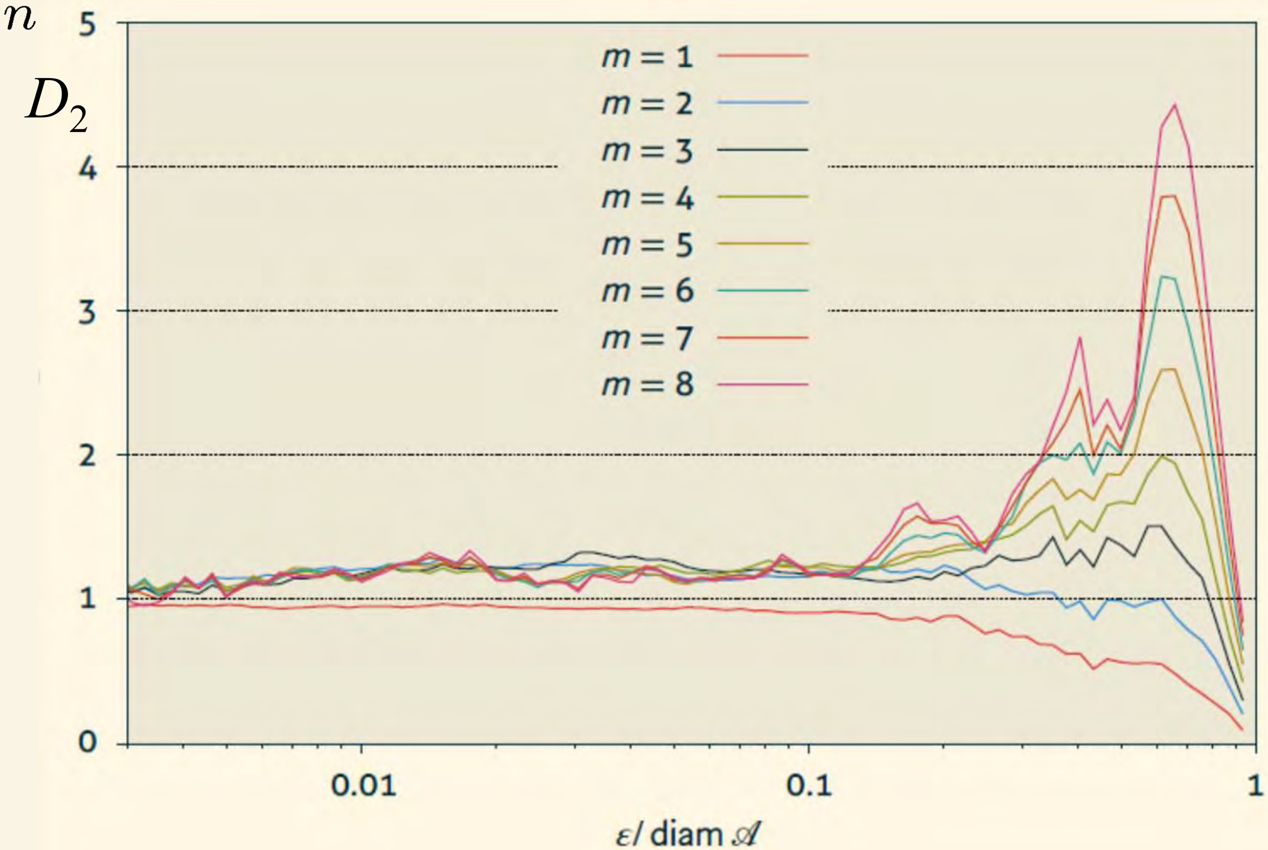


literature:
$D_2 \sim 1.26$

from: G. Ansmann

Generalized dimensions Rényi dimensions

dimension from time series:

dimension depends on the scope

How many dimensional is a plate of spaghetti?
Zero when seen from a long distance,
two on the scale of the plate,
one on the scale of the individual noodles
and three inside a noodle.
Maccaroni is even worse.

attributed to Peter Grassberger

Generalized dimensions Rényi dimensions

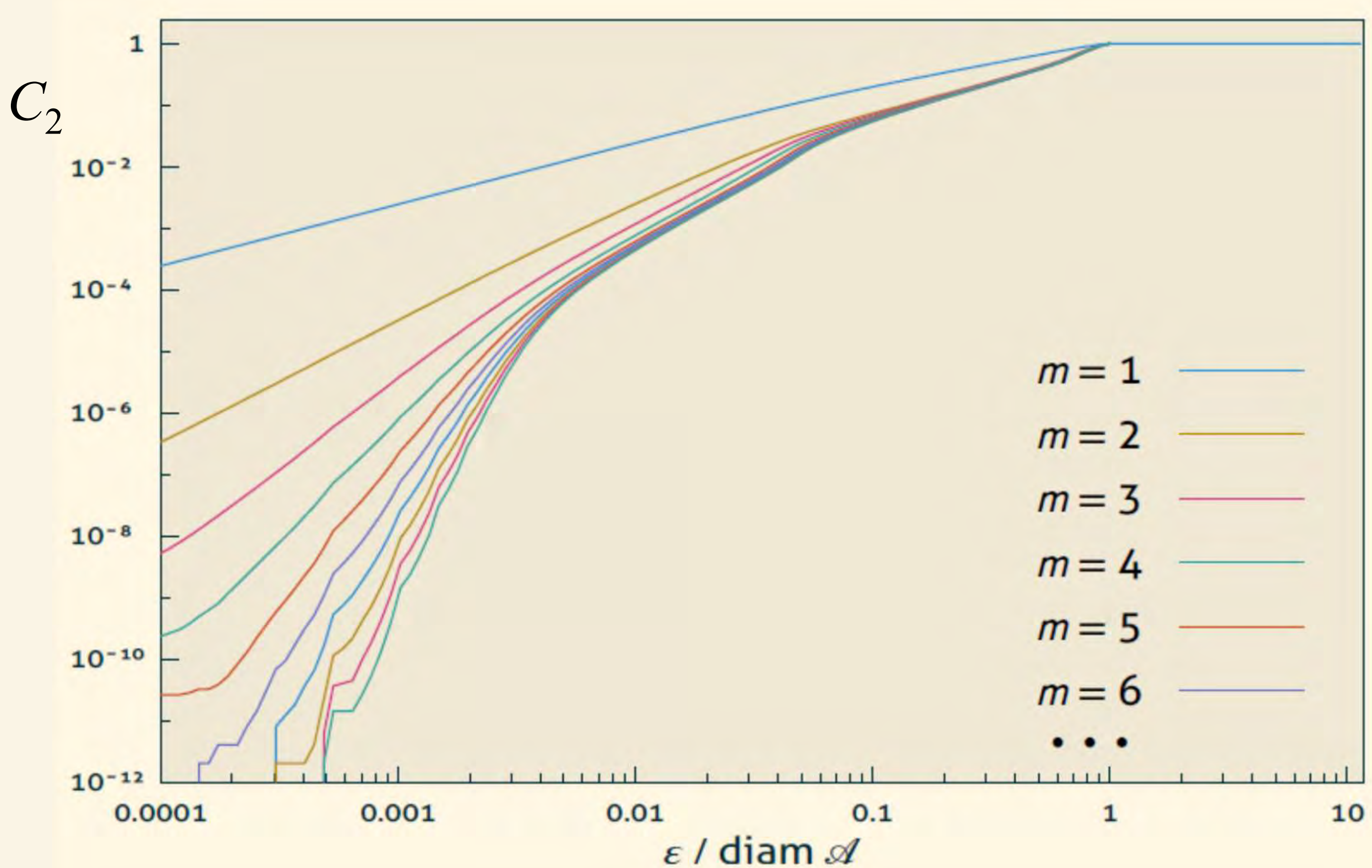
dimension from time series: example: torus

simulating maccaroni: sum of two incommensurable sine waves (tube/torus) and some noise (dough):

from: G. Ansmann

Generalized dimensions Rényi dimensions

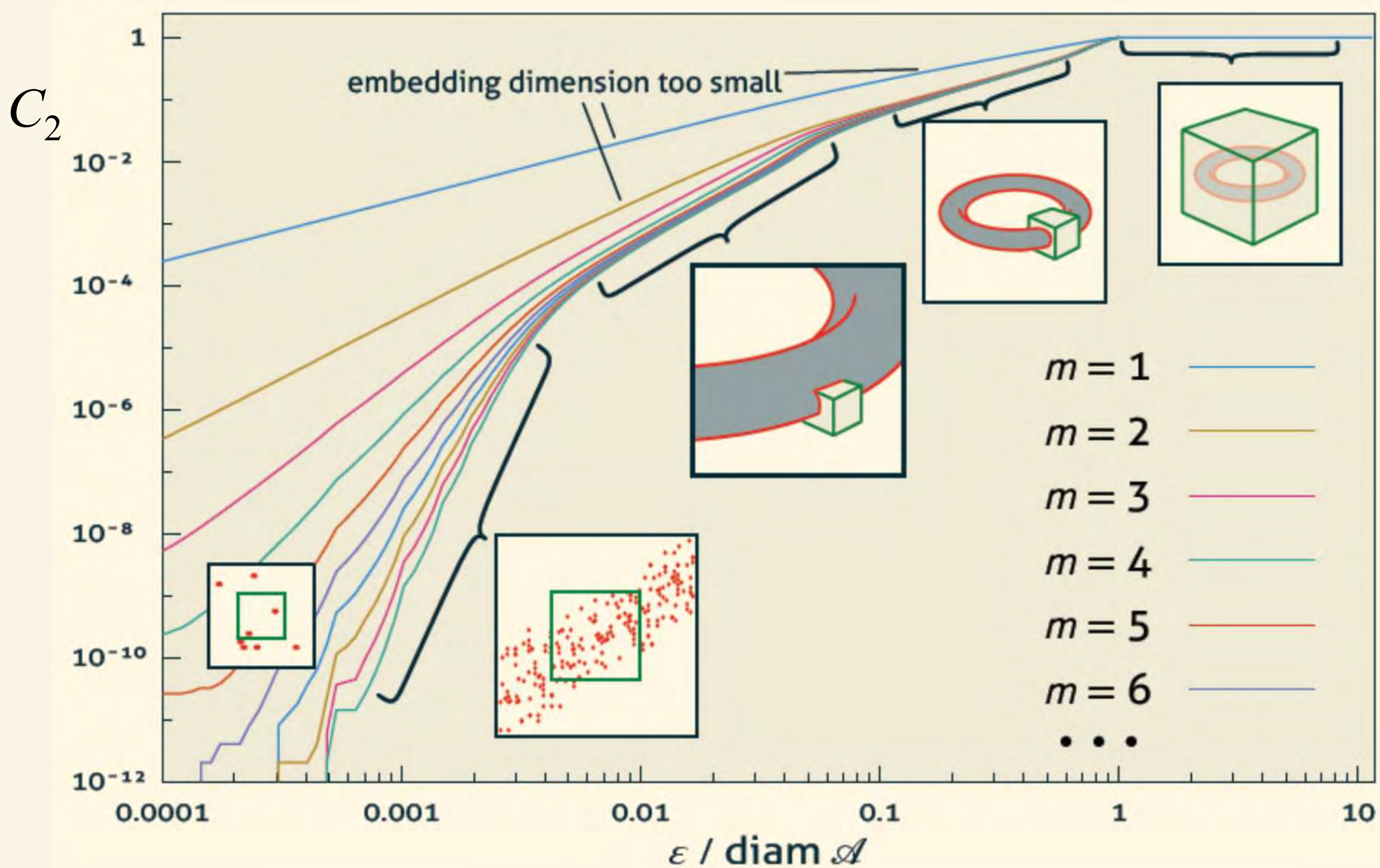
dimension from time series: example: torus



from: G. Ansmann

Generalized dimensions **Rényi dimensions**

dimension from time series: **example: torus**



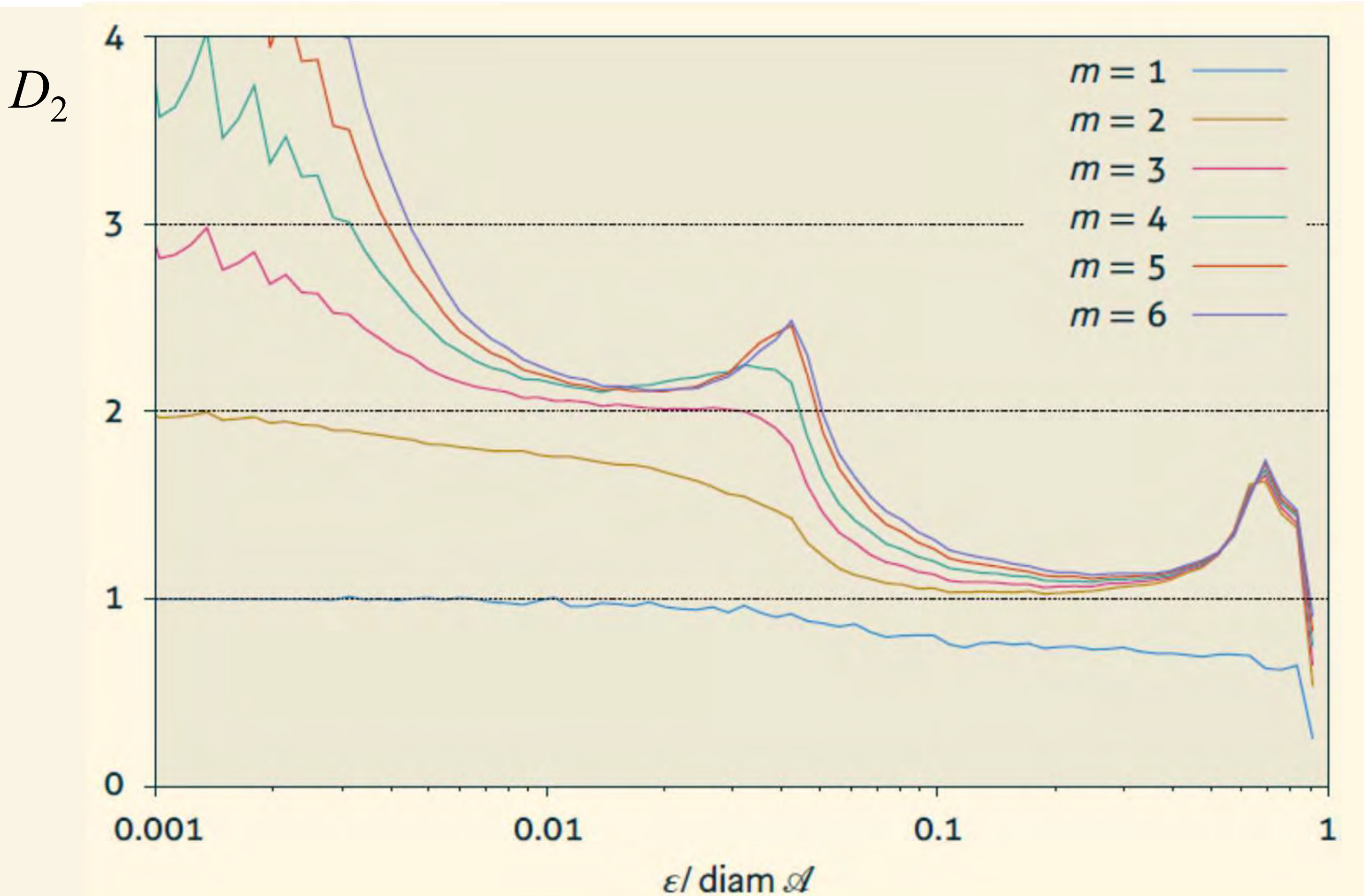
from: G. Ansmann

Generalized dimensions

Rényi dimensions

dimension from time series:

example: torus



from: G. Ansmann

Generalized dimensions **Rényi dimensions**

dimension from time series: **what can go wrong?**

field applications

- number of data points ($\lim N \to \infty$)
- data precision ($\lim \varepsilon \to 0$)
- strong correlations in data (sampling interval)
- noise
- filtering
- superposition of non-interacting dynamical systems

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

number of data points

- requirement: $\lim N \to \infty$
- field applications: N always limited, stationarity issues, system life-time, observation time

- proposed estimators:
  $N \sim 10^{D_2}$ (Albano et al., 1987)
  $N \sim 42^{D_2}$ (Smith, 1988)
  $N \sim 100^{D_2}$ (Procaccia, 1989)

- N as large as possible; resolvability of attractor structure depends on density of phase space points

Generalized dimensions Rényi dimensions

dimension from time series: what can go wrong?

number of data points determines maximum resolvable dimension (Ruelle criterion)

that if the slope in the Grassberger–Procaccia algorithm is measured over at least one decade one finds necessarily

$$\text{correlation dimension} \leqslant 2\log_{10} N$$

(with a base 10 logarithm). Indeed, the slope of interest is

$$(\log_{10} N(r'') - \log_{10} N(r'))/(\log_{10} r'' - \log_{10} r'),$$

where

$$r_{\min} \leqslant r' < r'' \leqslant r_{\max}$$

and therefore

$$N(r') \geqslant 1,$$
$$N(r'') \leqslant \tfrac{1}{2}N(N-1) < N^2,$$
$$\log_{10} N(r'') - \log_{10} N(r') \leqslant \log_{10} N^2.$$

But because $r'' \geqslant 10r'$ we have also

$$\log_{10} r'' - \log_{10} r' \geqslant \log_{10} 10$$

and therefore

$$\text{slope} \leqslant 2\log_{10} N$$

from: D. Ruelle. The Claude Bernard Lecture, 1989 - Deterministic chaos: the science and the fiction. Proc. R. Soc. Lond. A 427, 241-248 (1990)

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

Along the lines of the 'science and fiction' title of this talk, let me conclude on a lighter note. Readers of '*The hitchhiker's guide to the galaxy*', that masterpiece of British literature by D. Adams, know that a huge supercomputer has answered 'the great problem of life, the universe, and everything'. The answer obtained after many years of computation is 42. Unfortunately, one does not know to what precise question this is the answer, and what to make of it. It think that what happened is this. The supercomputer took a very long time series describing all it knew about 'life, the universe, and everything' and proceeded to compute the correlation dimension of the corresponding dynamics, using the Grassberger–Procaccia algorithm. This time series had a length N somewhat larger than 10^{21}. And you can imagine what happened. After many years of computation the answer came: dimension is approximately $2\log_{10} N \approx 42$.

from: D. Ruelle. The Claude Bernard Lecture, 1989 - Deterministic chaos: the science and the fiction. Proc. R. Soc. Lond. A 427, 241-248 (1990)

Generalized dimensions Rényi dimensions

dimension from time series: what can go wrong?

data precision

- field applications: analog-digital converter (ADC with *n* bits)
- digitizing accuracy: $p = A/2^n$, A = amplitude range
- quantization error: $q = p/2$

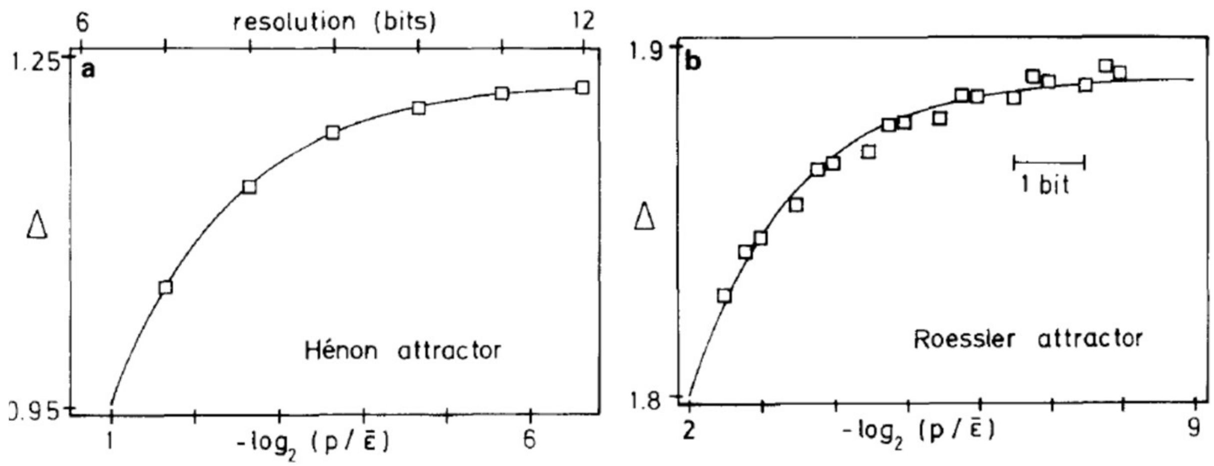


Fig. 1. Slopes of log–log plots of the correlation integral (estimates of D_2) calculated for data rounded to different resolutions, for (a) the Hénon attractor, (b) the Roessler attractor. In (a) one scaling region and in (b) three slightly different scaling regions were evaluated (digitizing accuracy p, average length scale $\bar{\epsilon}$).

The values of Δ are calculated by a least squares fit of the slope of the $\log C(\epsilon)$ versus $\log \epsilon$ plots over a selected scaling region $\epsilon_{min} \leqslant \epsilon \leqslant \epsilon_{max}$ (represented by an average length scale $\bar{\epsilon} \equiv (\epsilon_{min}\epsilon_{max})^{1/2}$). The data can be well fitted by the equation

$$\Delta = D_2(1 - kp/\bar{\epsilon}) , \qquad (2)$$

where p is one half of the least significant digit and k is a positive factor of order unity.

from: M. Möller, et al. *Errors from digitizing and noise in estimating attractor dimensions. Phys. Lett. A, 138(4-5), 176–182, 1989*

Generalized dimensions Rényi dimensions

dimension from time series: what can go wrong?

data precision

possible way to minimize influence of the quantization error:
add noise prior to digitizing
(pre-whitening, dithering, bleaching)

effectiveness depends on system under study
not effective for broad-band signals

caveat: adding noise can lead to erroneous dimension estimates

from: M. Möller, et al. *Errors from digitizing and noise in estimating attractor dimensions. Phys. Lett. A, 138(4-5), 176–182, 1989*

Generalized dimensions Rényi dimensions

dimension from time series: what can go wrong?

strong correlations in data

field applications

- sampling rate according to Nyquist-Shannon theorem:

  at least twice as high as signal's maximum frequency $f_{\max}$ *to avoid aliasing*

- how to treat cases with unknown $f_{\max}$?
- how to treat a chaotic signal?
- is resampling (over-/undersampling) a good choice?

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

strong correlations in data

problem: for a sufficiently fine temporal resolution, points close in time are also close in phase space

→ correlation sum overestimated

Theiler correction:

exclude temporally close points from the correlation sum:

$$\sum_{i,j} \Theta\left(\epsilon - \left|\vec{v_i} - \vec{v_j}\right|\right) \quad \rightarrow \quad \sum_{|i-j|>W} \Theta\left(\epsilon - \left|\vec{v_i} - \vec{v_j}\right|\right)$$

(adjust normalization accordingly)

J. Theiler, Phys. Rev. A, 34(3):2427-2432 1986

Generalized dimensions **Rényi dimensions**

dimension from time series: **what can go wrong?**

strong correlations in data

how to choose cutoff W for Theiler correction?

minimum requirement: W in the order of autocorrelation time (Δ)

better: $$W > \Delta \left(\frac{2}{N}\right)^{\frac{2}{m}}$$

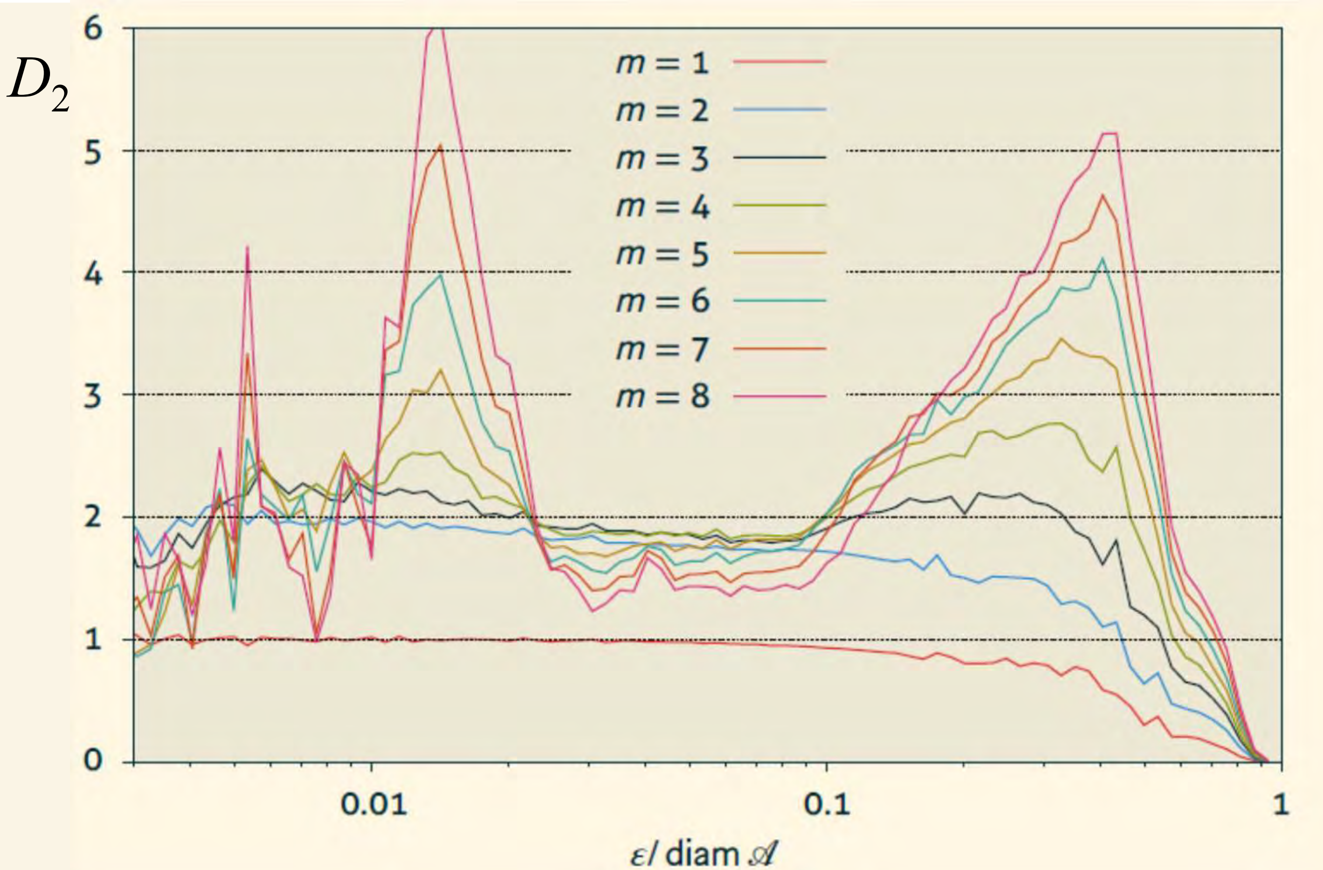
(exact choice is then insignificant)

J. Theiler, Phys. Rev. A, 34(3):2427-2432 1986

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

strong correlations in data (ex.: Lorenz system without correction)

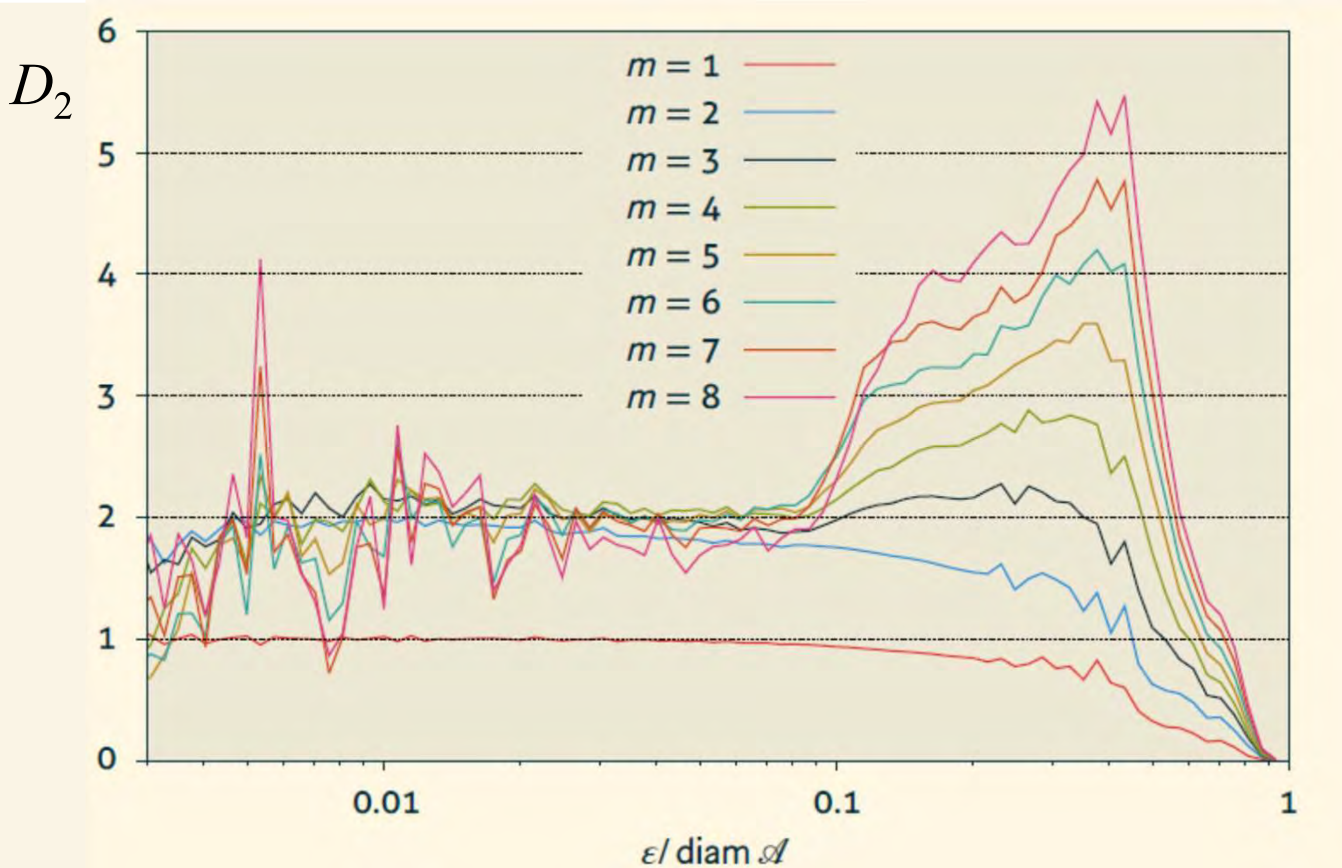


from: G. Ansmann

Generalized dimensions **Rényi dimensions**

dimension from time series: **what can go wrong?**

strong correlations in data (ex.: Lorenz system with correction)



from: G. Ansmann

Generalized dimensions Rényi dimensions

dimension from time series: what can go wrong?

noise

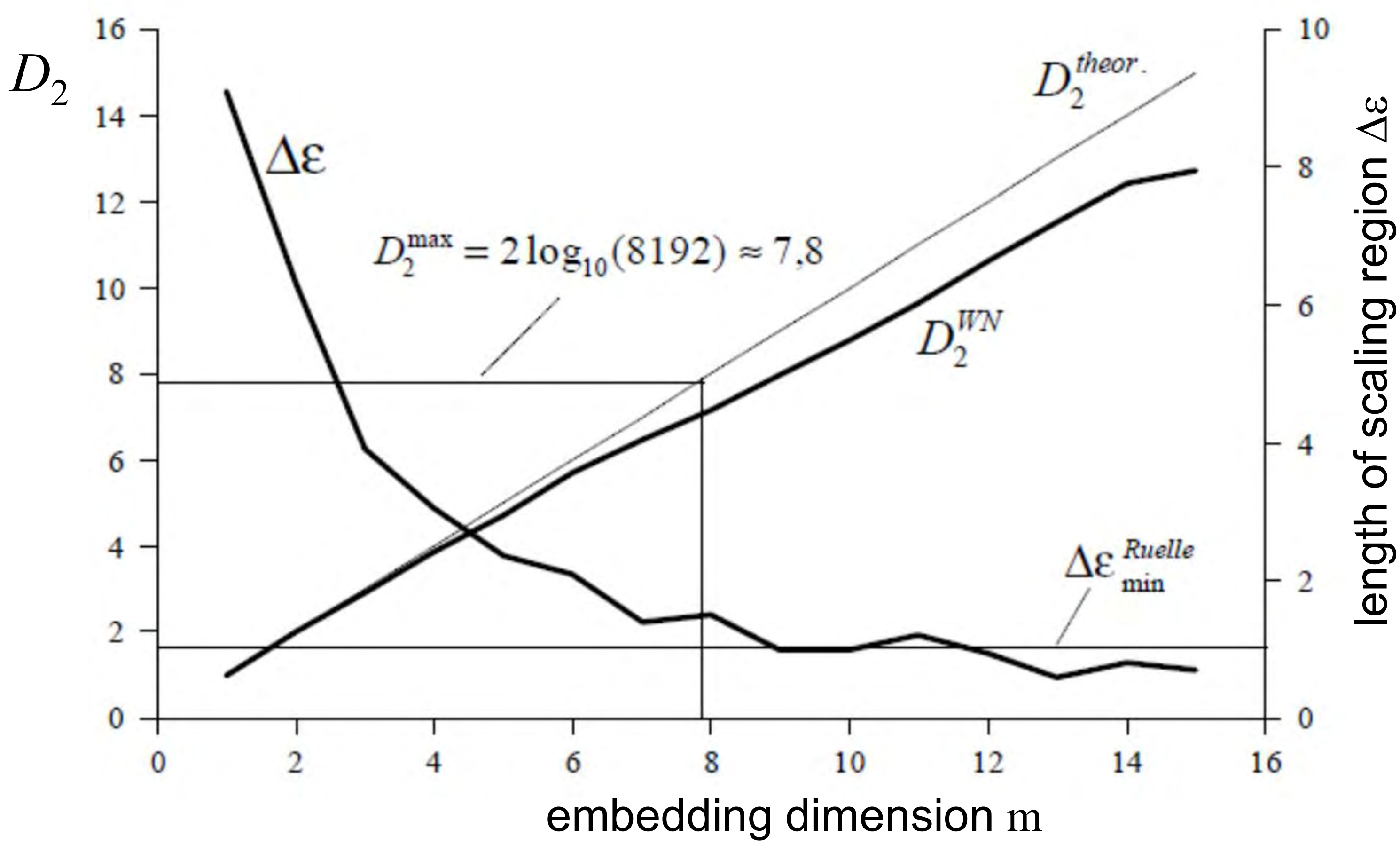
field applications:

- data always noisy (characteristics of noise?)
- measurement errors (white noise approximation)
- additive vs. multiplicative noise
- if number of data points limited, dimension of white noise finite!

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

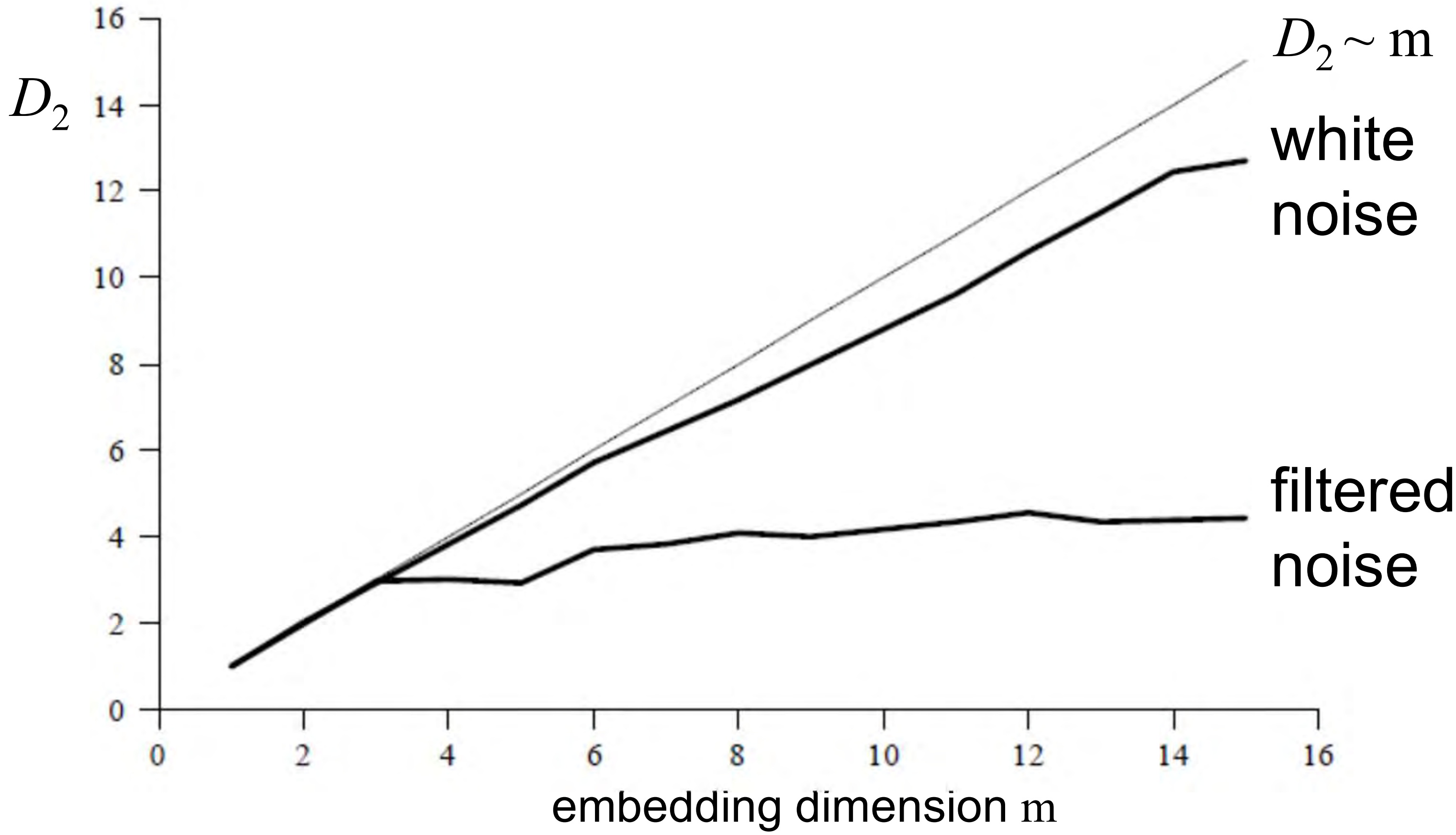
example: white noise; N = 8192



Generalized dimensions **Rényi dimensions**

dimension from time series: **what can go wrong?**

example: low-pass-filtered noise; N = 8192, Theiler correction

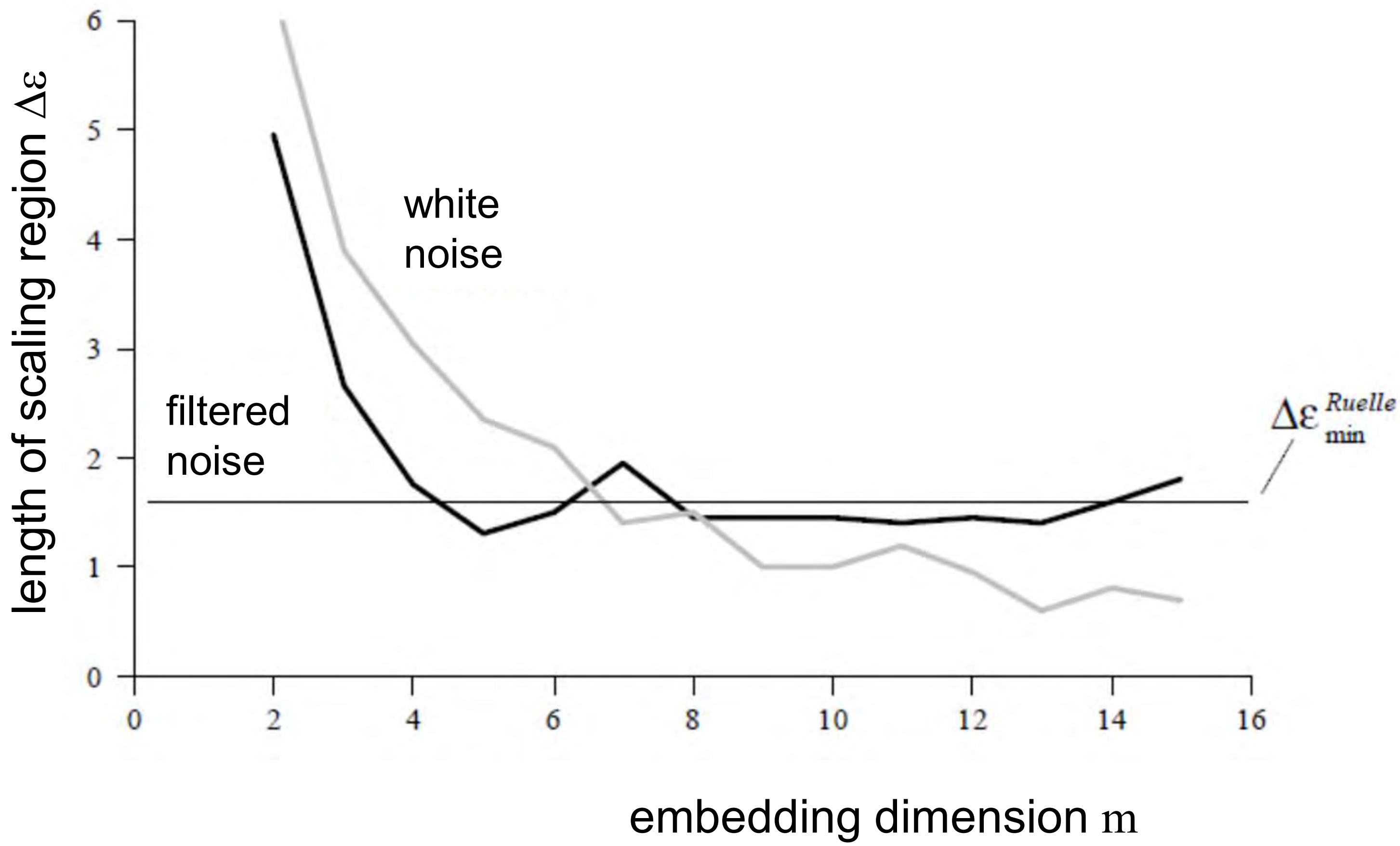


Rapp et al., Filtered noise can mimic low-dimensional chaotic attractors. Phys. Rev. E, 47, 2289 (1993)

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

example: low-pass-filtered noise; N = 8192, Theiler correction



Generalized dimensions **Rényi dimensions**

dimension from time series: **what can go wrong?**

noise

classical filtering of noise induces structure in phase space
→ spatial (long-ranged) correlations

Theiler correction only minimized short-ranged correlations

$\Rightarrow$
- do not use classical filter for chaotic signals !
- apply other methods to discriminate determinism from stochasticity
- use other nonlinear noise reduction schemes (future lectures)

Generalized dimensions Rényi dimensions

dimension from time series: what can go wrong?

filtering

field applications:
- sampling theorem, avoid aliasing
- noise reduction (see above!)

- chaotic signals typically broad-band (see Linear Methods)
- do not filter chaotic signals !

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

example: filtered Hénon map

$$x_{n+1} = 1 - a x_n^2 + y_n$$
$$y_{n+1} = b x_n \qquad \text{(a=1,4; b=0,3)}$$
$$z_{n+1} = \exp(-\eta) z_n + x_n$$

recursive realization of single-pole low-pass filter (1. order)

$$\dot{z} = -\eta z + x$$

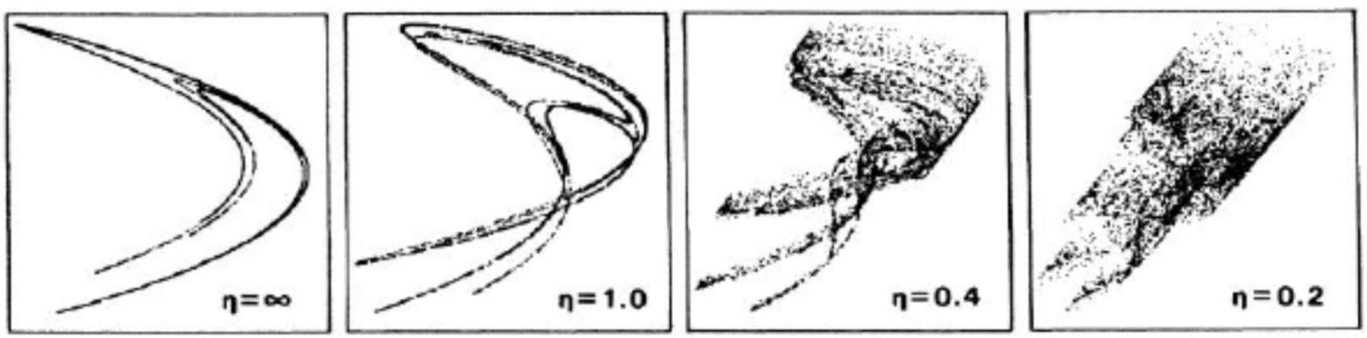


FIG. 3. Influence of degree of filtering on attractor shape. Shown are return maps (Z_{n+1} vs Z_n) of the "filtered Hénon system," Eq. (4), with eight-bit resolution. All four attractors are scaled to the same size.

Badii et al., Phys. Rev. Lett. 60, 979, 1988; Mitschke et al., Phys Rev A 37, 4518, 1988)

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

example: filtered Hénon map

dimension increase (+1)

interpretation:
- superposition of two systems (Hénon system + filter)
- filter (passive, linear) one-dimensional system

filtering does not affect other invariant measures

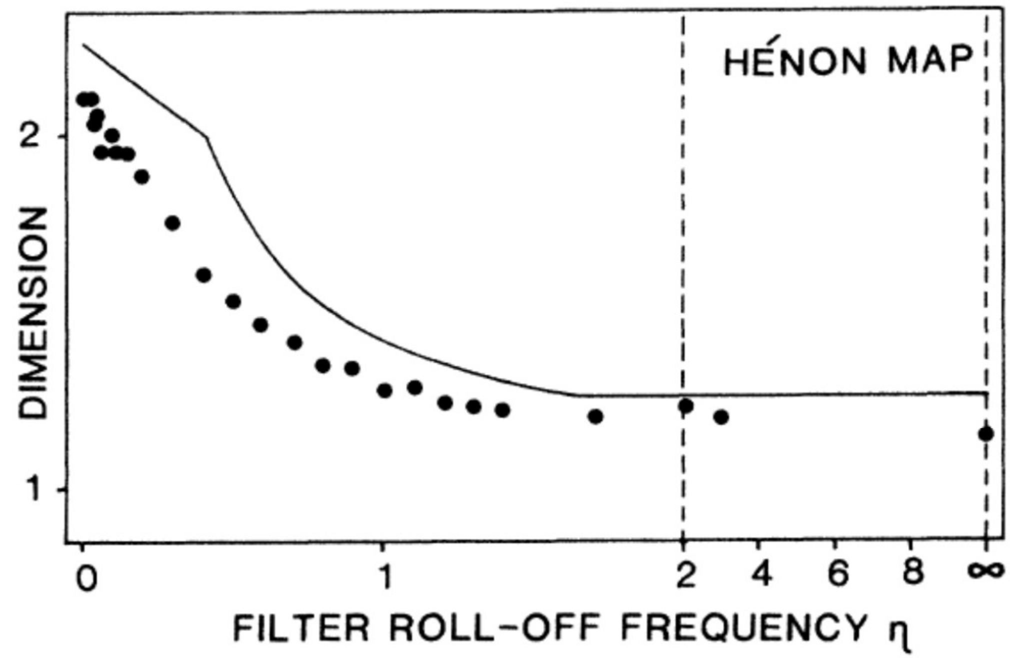


FIG. 1. D_2 values for simulated low-pass filtering with different rolloff frequencies η for the Hénon map. The solid line shows the predicted $D_1(\eta)$. Note change of scale at the dashed lines.

Badii et al., Phys. Rev. Lett. 60, 979, 1988; Mitschke et al., Phys. Rev. A 37, 4518, 1988

Generalized dimensions Rényi dimensions

dimension from time series: what can go wrong?

product dynamical system

- Consider two non-interaction dynamical systems ***A*** and ***B***. For the *product dynamical system* ***A***x***B***, we have

$$dim(\mathbf{A}\times\mathbf{B}) \leq dim(\mathbf{A}) + dim(\mathbf{B}).$$

- Consider time series of system observables *v*(***A***), resp. *v*(***B***) that solely depend on ***A***, resp. ***B***, and a time series *v*(***A***x***B***) of the product dynamical system with

$$v(\mathbf{A}\times\mathbf{B}) = \alpha\, v(\mathbf{A}) + \beta\, v(\mathbf{B})$$

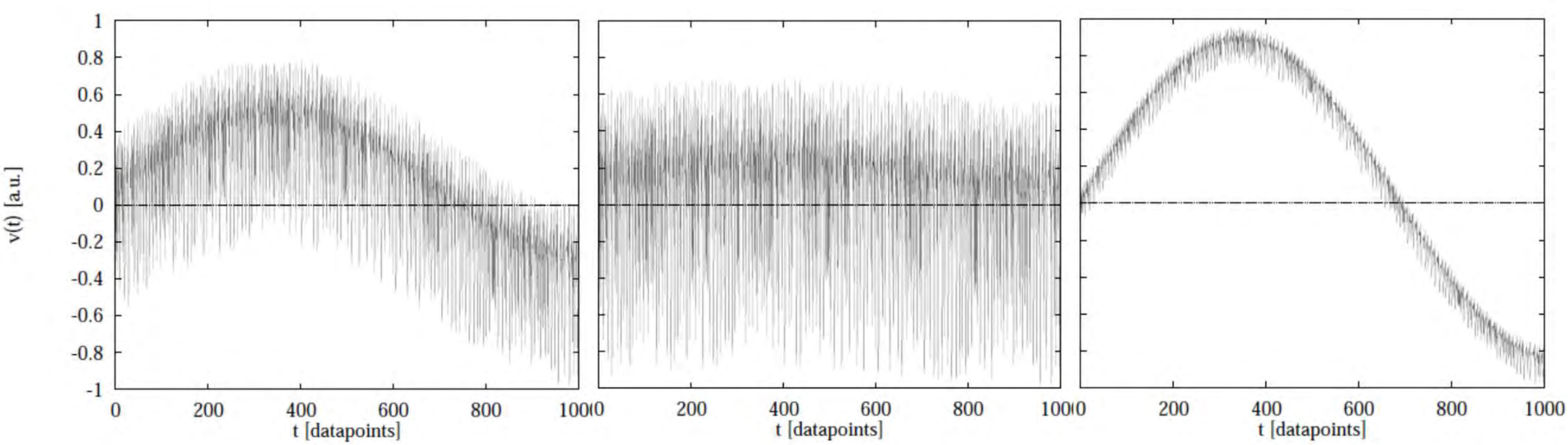
- Consider cases $\alpha = \beta$, $\alpha < \beta$, and $\alpha > \beta$

Hurewicz and Wallmann (1948): Dimension Theory

Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

product dynamical system

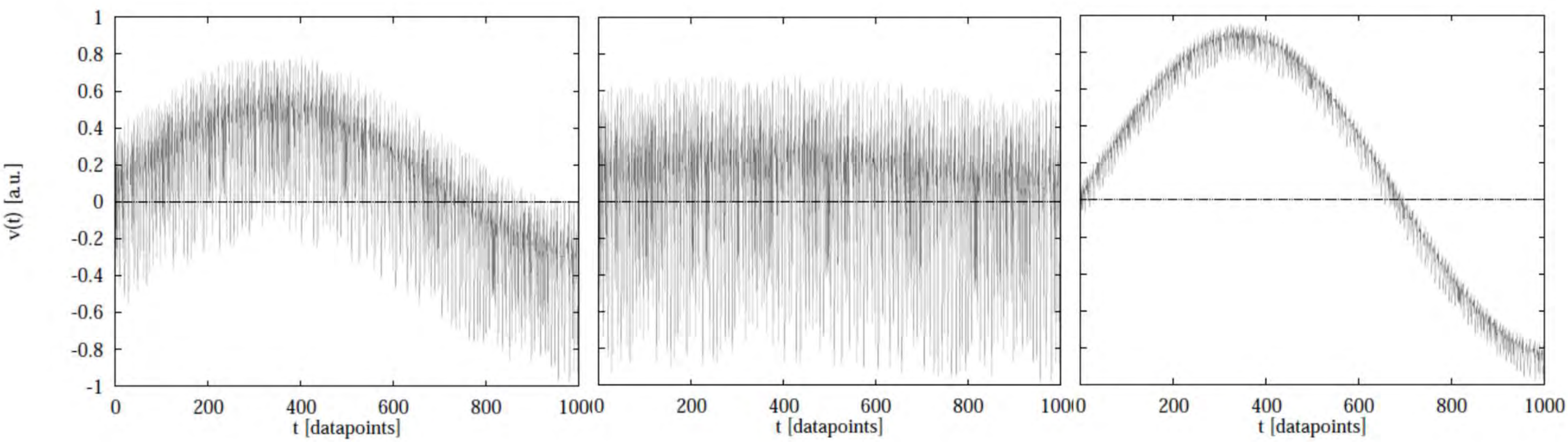


Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

product dynamical system

sine wave (***A***) + Hénon map (***B***)

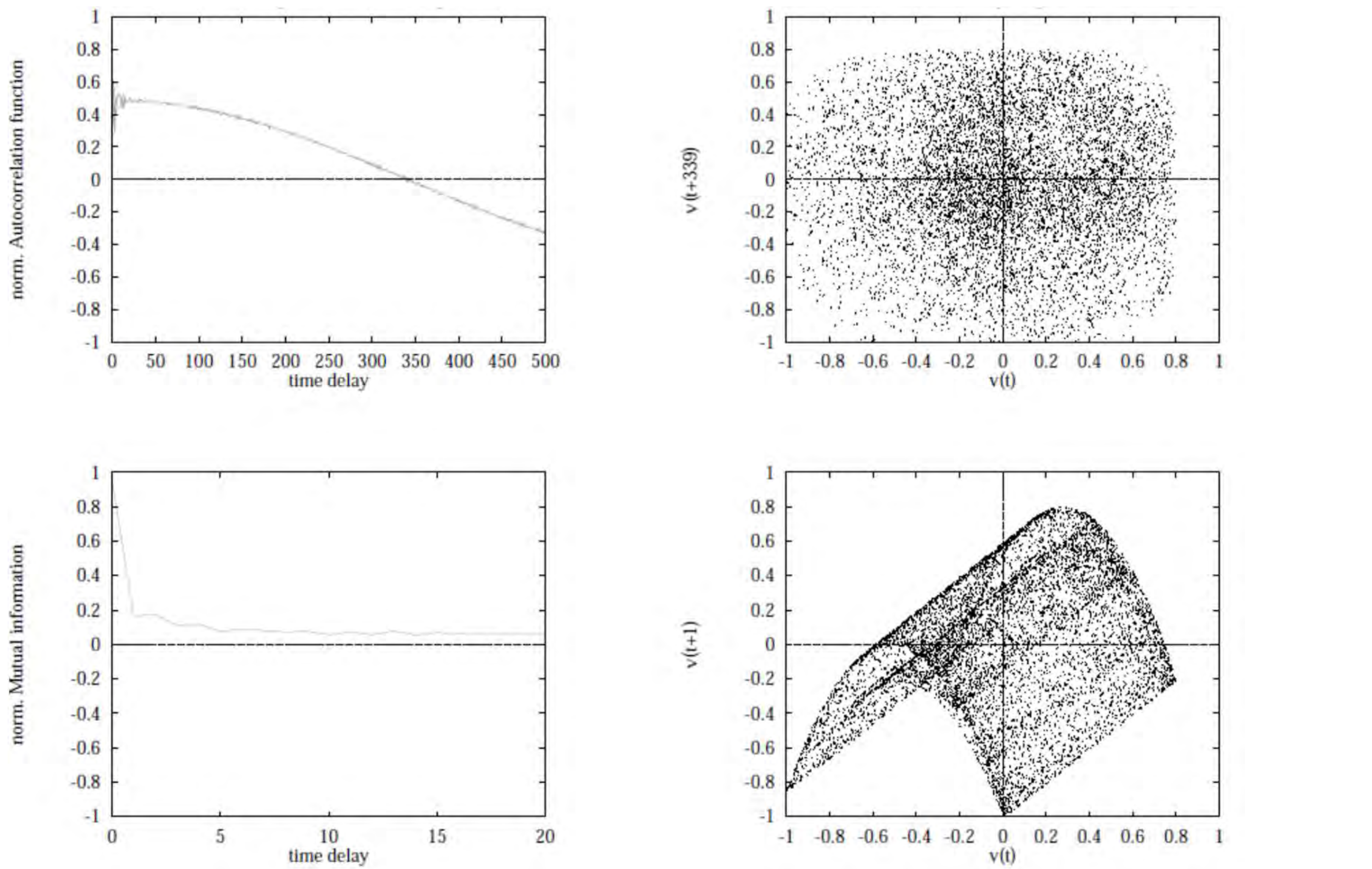


$\alpha = \beta = 1$ $\alpha = 0.1; \beta = 1$ $\alpha = 1; \beta = 0.1$

Generalized dimensions Rényi dimensions

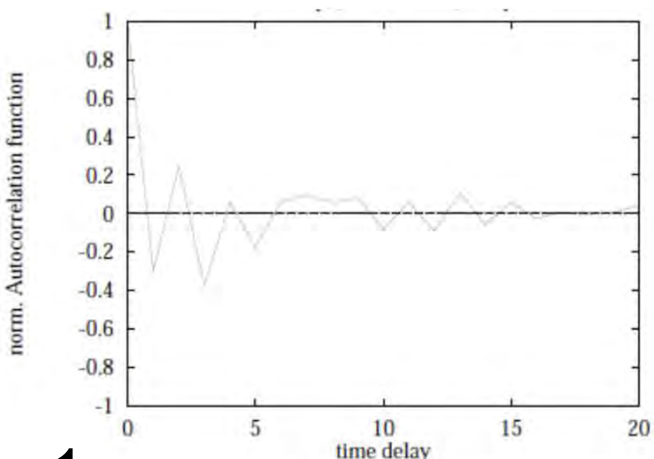
dimension from time series: what can go wrong?

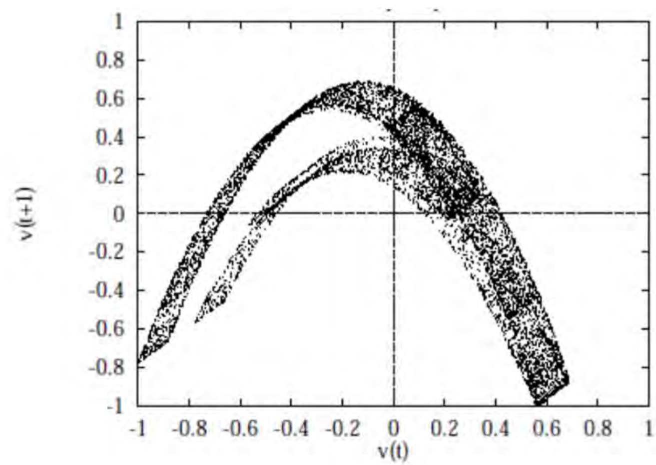
$\alpha = \beta = 1$



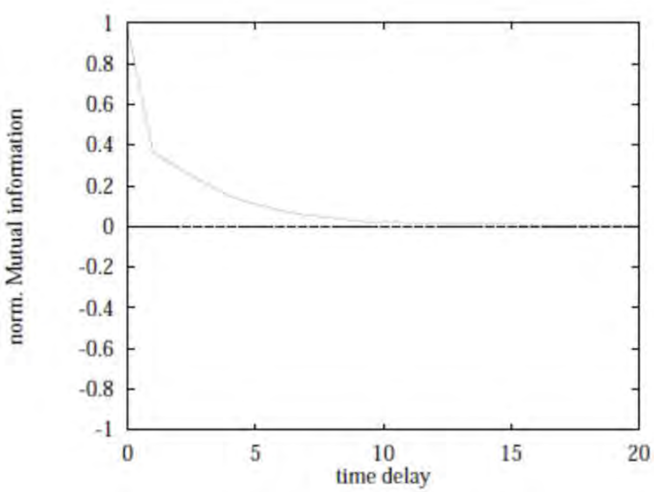
Generalized dimensions Rényi dimensions

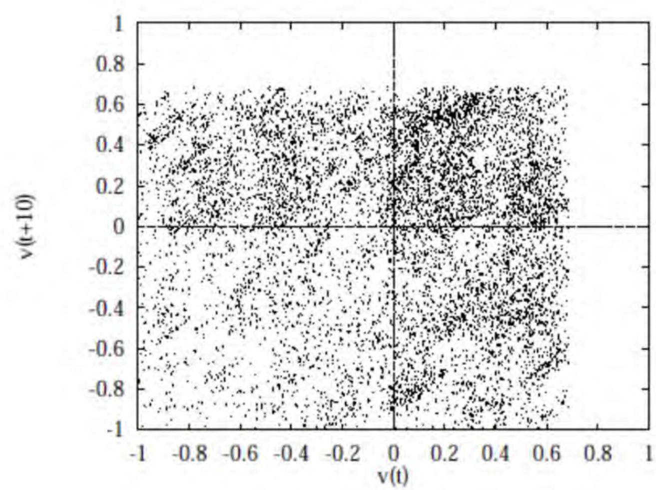
dimension from time series: what can go wrong?





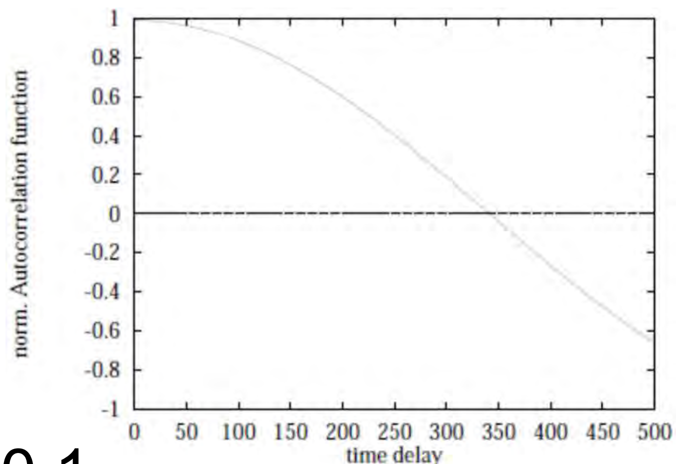
$\alpha = 0.1; \beta = 1$

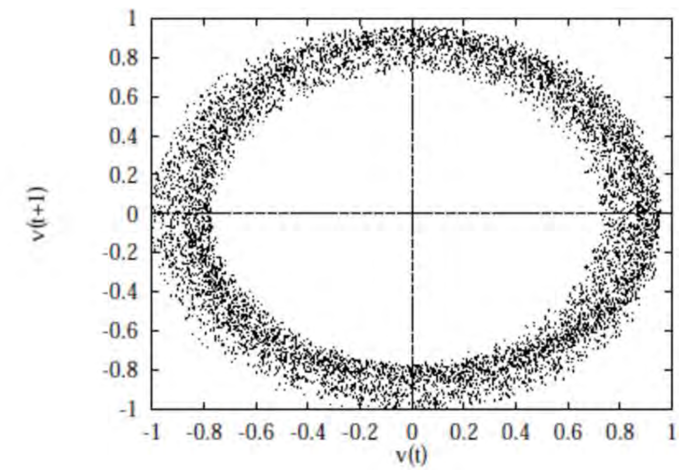




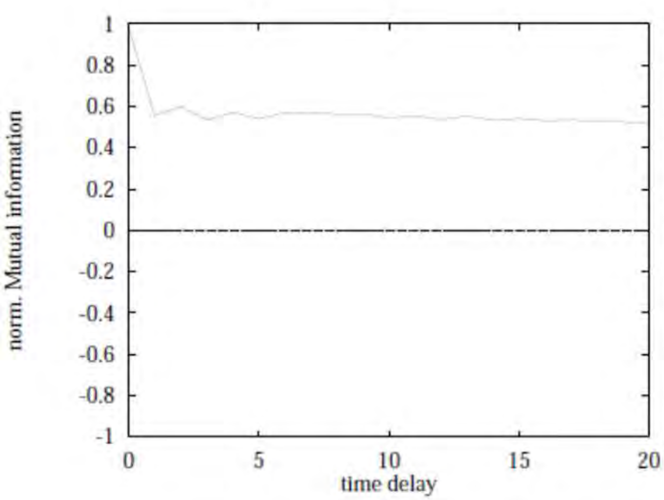
Generalized dimensions Rényi dimensions

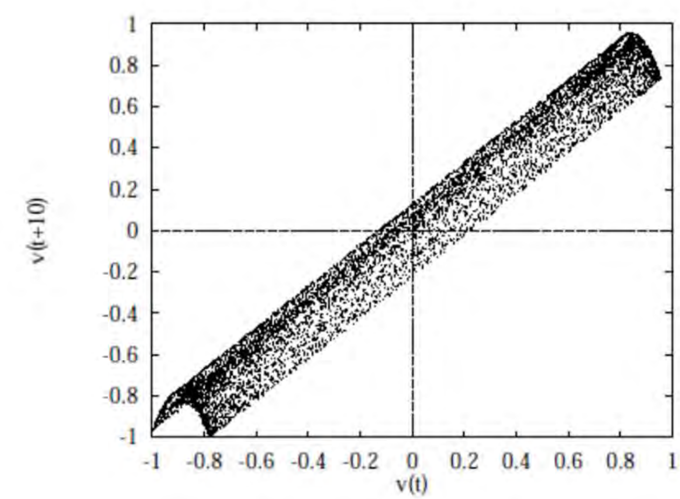
dimension from time series: what can go wrong?





$\alpha = 1; \beta = 0.1$



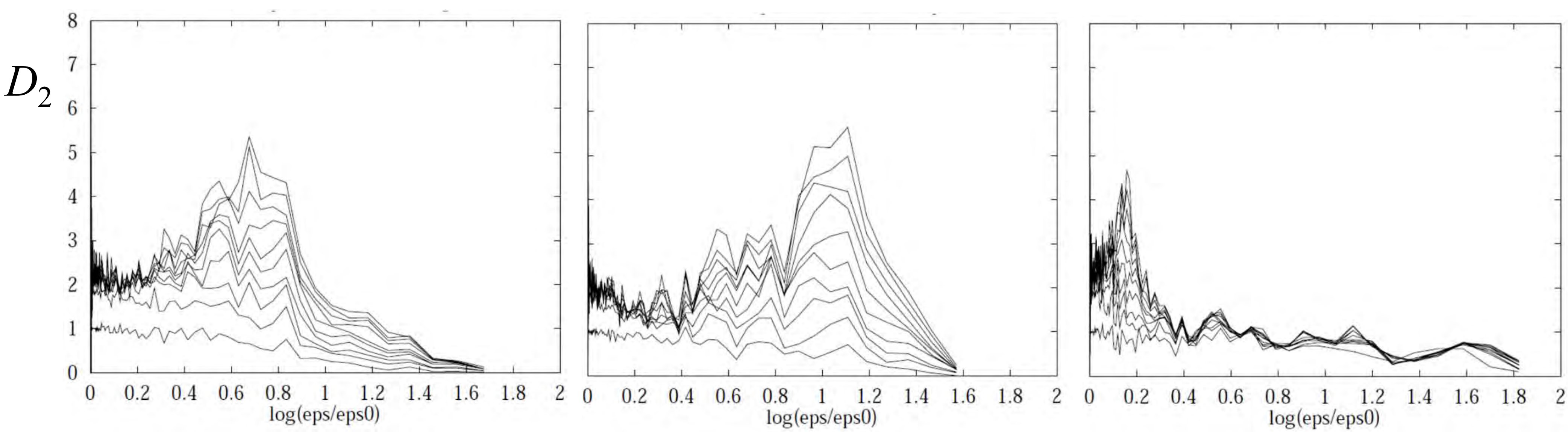


Generalized dimensions — Rényi dimensions

dimension from time series: what can go wrong?

product dynamical system

sine wave (***A***) + Hénon map (***B***)



$\alpha = \beta = 1$ $\alpha = 0.1;\ \beta = 1$ $\alpha = 1;\ \beta = 0.1$

Generalized dimensions **Rényi dimensions**

dimension from time series: **Summary**

Estimate dimension of time series via slope of correlation sum

- check multiple embedding dimensions *m*
- select scaling region properly
- apply Theiler correction
- be aware of influencing factors, limitations, and pitfalls

$D \notin \mathbb{N}$: chaotic dynamics

$D \in \mathbb{N}$: hint at regular dynamics

$D = \infty$: noise