univariate nonlinear time series analysis

- most systems are inherently nonlinear in nature
  (linear systems are rare)

- nonlinear dynamical systems can exhibit rich dynamical behaviors
  (bifurcations, chaotic motion, amplitude death, multistability, …)

- linear time series analysis generally fail to fully characterize
  nonlinear dynamics
  (*looks like noise*)

- alternative approach: characterize dynamics in phase space
  (from the time domain to a geometrical description)

univariate nonlinear time series analysis

- Takens' theorem allows reconstruction of a topologically equivalent phase space even from a single time series (delay embedding)

- embedding parameters (embedding delay, embedding dimension) can be identified in a data-driven way (although not unique)

- dynamical invariants (dimensions, Lyapunov exponents, entropies) allow comprehensive characterization of dynamics in phase space

- dimensions can characterize scaling behavior, self-similarity, number of degrees of freedom, complexity, determinism, nonlinearity

- Lyapunov exponents can characterize stability (short- and long-term), predictability, determinism, nonlinearity, chaoticity

univariate nonlinear time series analysis

- entropies can characterize (dis-)order, complexity, predictability, determinism, nonlinearity, chaoticity

- all dynamical invariants can distinguish between regular, chaotic, and stochastic motion

- dynamical invariant are related to each other (Kaplan-Yorke conjecture, Pesin's identity)

- definitions of dynamical invariants are based on prerequisites that can often not be fulfilled in a strict sense when analyzing field data

- important assistant techniques: testing for determinism, testing for nonlinearity, and nonlinear noise reduction

univariate nonlinear time series analysis

- testing for determinism: evaluates the smoothness of local flow in phase space to better distinguish between (nonlinear) deterministic and stochastic dynamics

- testing for nonlinearity: generate surrogate time series that have the same (statistical) properties as the original time series, except nonlinearity; various concepts for surrogate generation; compare characteristics of original time series (e.g. via dynamical invariants) with the surrogates' ones using null hypothesis testing and robust test statistics.

- nonlinear noise reduction (caveat: shadowing problem) in phase space; dynamical methods (function approximation, averaging) and geometric methods (projection onto lower-dimensional manifold)

univariate nonlinear time series analysis

- many influencing factors
 (related to system, environment, data acquisition, analysis)

- blind and improper application of nonlinear time series analysis techniques can lead to severe misinterpretations

- understanding the impact of influencing factors can help to avoid errors and misinterpretations, and can help to improve analysis techniques and develop novel ones

- univariate analysis techniques can not characterize relationships between (sub-)systems

Measuring Interactions from Time Series

Introduction and Statistical Techniques

why is analyzing interactions of interest?

- time series of different observables from the same dynamical system
  redundancies, mutual influences, …

- time series of different/same observables from subsystems (if system can be decomposed into different subsystems)
  redundancies, mutual influences, couplings, synchronization,
  causal relationships, stability, (cross-)prediction, …
  (e.g. interactions between different organ system →
  emerging scientific field: *network physiology*)

- time series of observables from different dynamical systems
  mutual influences, couplings, synchronization,
  causal relationships, stability, (cross-)prediction, …

understanding interactions

Qs:
- is there an interaction?
- if so,
  - how strong is it?
  - are there directional (causal) relationships?
  - how does the coupling function look like?

As:
- can be derived from ***active*** experiments
  (probing; *actio est reactio* (Newton))
- study responses of systems to some perturbation
  (relaxation phenomena)

understanding interactions

Qs:

- active experiments are not possible,
  but can one still analyze interactions?

As:

- use bivariate analysis techniques to estimate characteristics of an interaction (strength, direction, coupling function*) from time series (passive observations)

* work in progress

understanding interactions additional literature

linear systems:

B. Everitt, Multivariate Techniques for Multivariate Data, North-Holland, 1978

nonlinear systems:

A. Pikovsky, M. Rosenblum, J. Kurths, Synchronization: A universal concept in nonlinear sciences . Cambridge University Press, 2001

S. Boccaletti, J. Kurths, G. Osipov, D.L. Valladares, C.S Zhou. The synchronization of chaotic systems. *Physics Reports*, 366, 1, 2002

T. Stankovski, T. Pereira, P.V. McClintock, A. Stefanovska. Coupling functions: universal insights into dynamical interaction mechanisms. *Reviews of Modern Physics*, 89, 045001, 2017

understanding interactions some definitions

consider two continuous-time dynamical systems X and Y described by:

$$\dot{\mathbf{x}} = f_{\mathrm{X}}(t, \mathbf{x}(t), \beta_{\mathrm{X}}); \quad \mathbf{x} \in \mathbb{R}^{d_{\mathrm{X}}} \ ; \ f_{\mathrm{X}} : \mathbb{R}^{d_{\mathrm{X}}} \to \mathbb{R}^{d_{\mathrm{X}}}$$

$$\dot{\mathbf{y}} = f_{\mathrm{Y}}(t, \mathbf{y}(t), \beta_{\mathrm{Y}}); \quad \mathbf{y} \in \mathbb{R}^{d_{\mathrm{Y}}} \ ; \ f_{\mathrm{Y}} : \mathbb{R}^{d_{\mathrm{Y}}} \to \mathbb{R}^{d_{\mathrm{Y}}}$$

coupling between X and Y can be regarded as generating a third system Z, with

$$\dot{\mathbf{z}} = \begin{cases} \dot{\mathbf{x}} = f^*_{\mathrm{X}}(t, \mathbf{x}(t), \beta_{\mathrm{X}}); & f^*_{\mathrm{X}} : \mathbb{R}^{d_{\mathrm{X}}+d_{\mathrm{Y}}} \to \mathbb{R}^{d_{\mathrm{X}}} \\ \dot{\mathbf{y}} = f^*_{\mathrm{Y}}(t, \mathbf{y}(t), \beta_{\mathrm{Y}}); & f^*_{\mathrm{Y}} : \mathbb{R}^{d_{\mathrm{X}}+d_{\mathrm{Y}}} \to \mathbb{R}^{d_{\mathrm{Y}}} \end{cases}$$

understanding interactions some definitions

uncoupled case:

system Z consist of two autonomous systems X and Y with their respective attractors

$$f^*_{\mathrm{X}} = f_{\mathrm{X}} \quad \text{and} \quad f^*_{\mathrm{Y}} = f_{\mathrm{Y}} \qquad \text{attractors } \mathcal{A}_{\mathrm{X}} \text{ and } \mathcal{A}_{\mathrm{Y}}$$

coupled case:

- coupling induces interactions between systems X and Y
- coupling “perturbs” eigen-dynamics of systems X and Y
- coupling leads to formation of attractor of system Z
- understanding coupling requires investigating the joint phase space

understanding interactions some definitions

diffusive coupling:

add "coupling term" of strength c and function $g_{\mathrm{X,Y}}(\cdot,\cdot)$ to eigen-dynamics:

$$\dot{\mathbf{z}} = \begin{cases} \dot{\mathbf{x}} = f^*_{\mathrm{X}}(t, \mathbf{x}(t), \beta_{\mathrm{X}}) = f_{\mathrm{X}}(t, \mathbf{x}(t), \beta_{\mathrm{X}}) + c_{\mathrm{X}} g_{\mathrm{X}}(\mathbf{x}, \mathbf{y}) \\ \dot{\mathbf{y}} = f^*_{\mathrm{Y}}(t, \mathbf{y}(t), \beta_{\mathrm{Y}}) = f_{\mathrm{Y}}(t, \mathbf{y}(t), \beta_{\mathrm{Y}}) + c_{\mathrm{Y}} g_{\mathrm{Y}}(\mathbf{y}, \mathbf{x}) \end{cases}$$

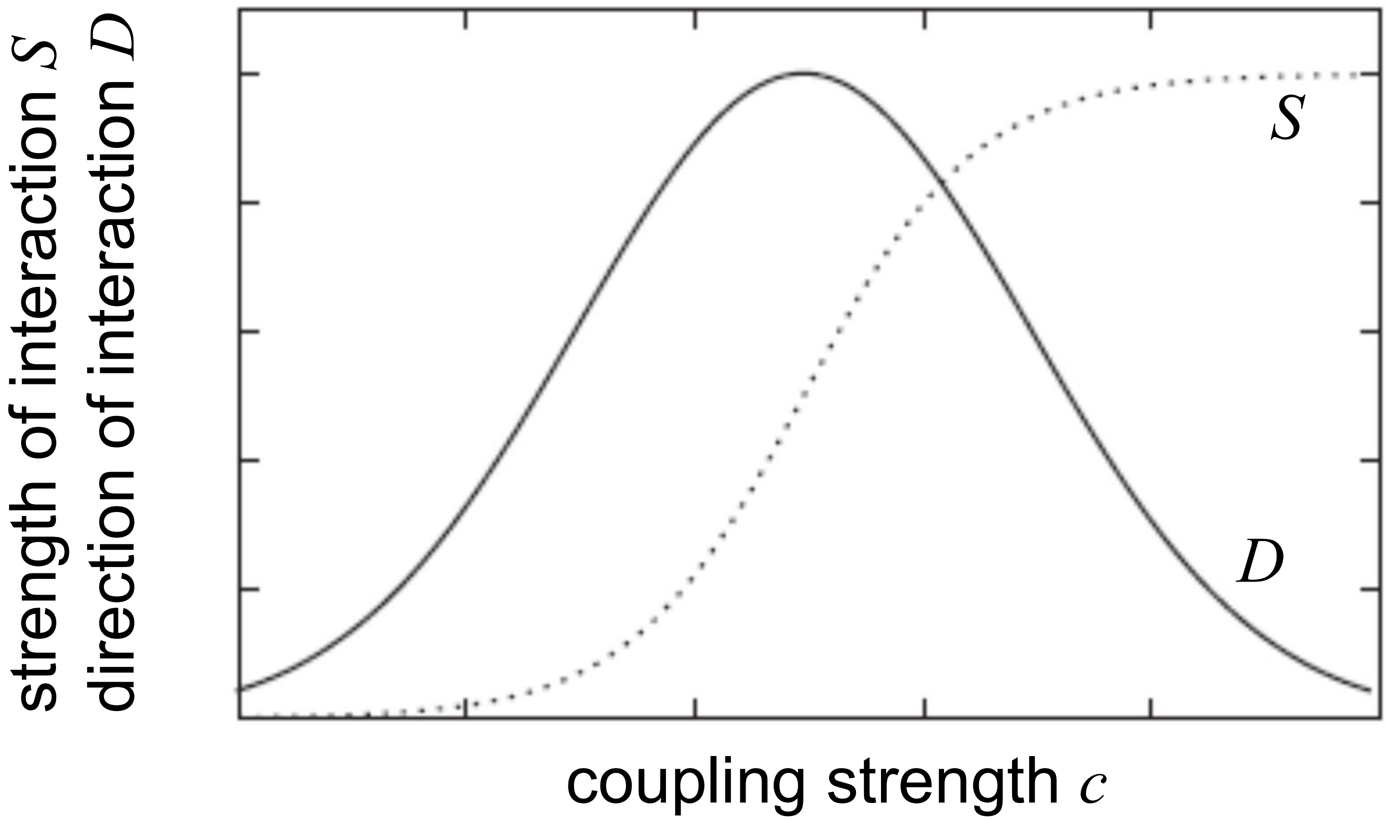
unidirectional (driver-responder / master-slave) coupling:

$$c_{\mathrm{X}} \neq 0 \vee c_{\mathrm{Y}} \neq 0$$

bidirectional coupling:

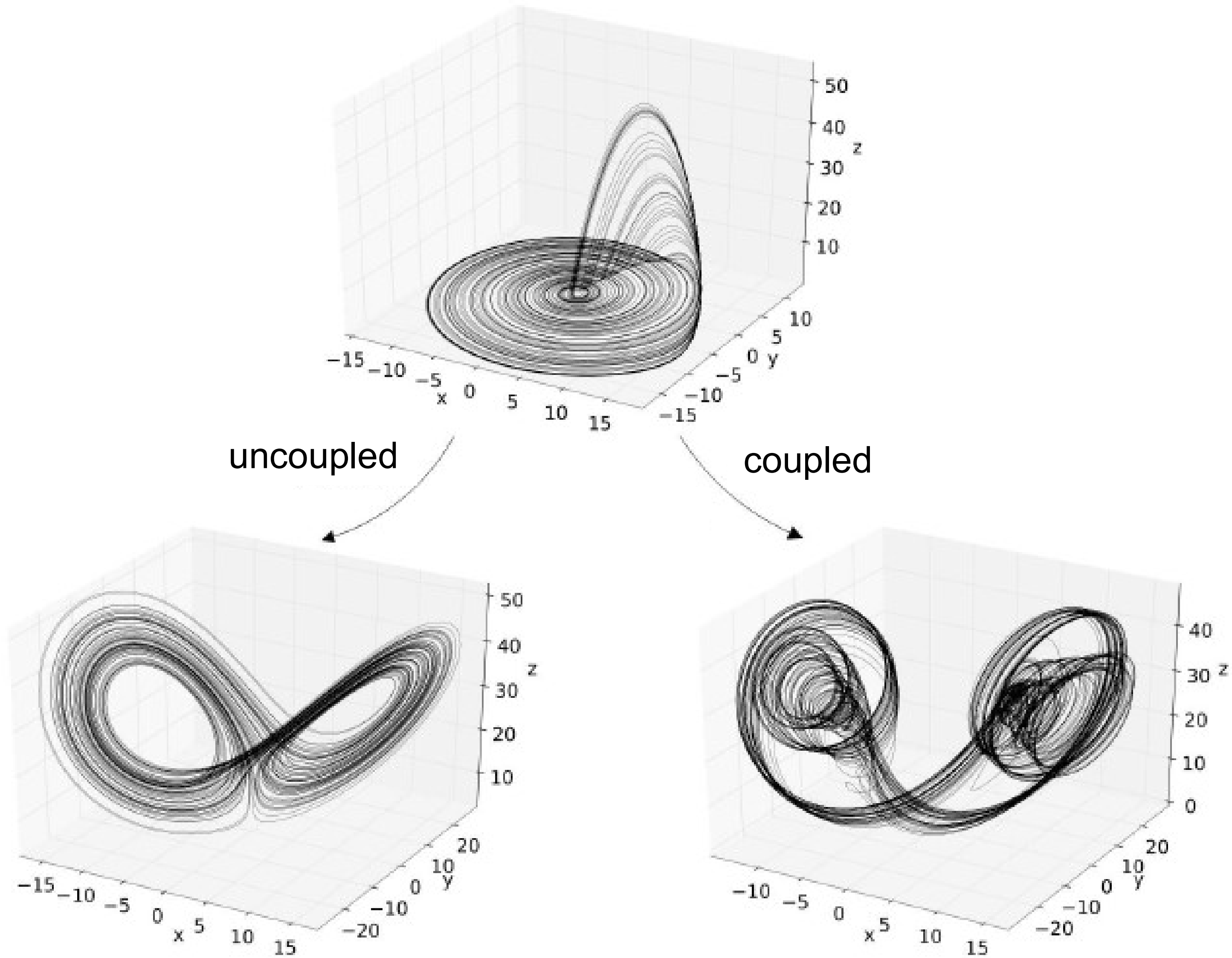
$$c_{\mathrm{X}} \neq 0 \wedge c_{\mathrm{Y}} \neq 0$$

measuring interactions some expectations



weak coupling regime strong coupling regime

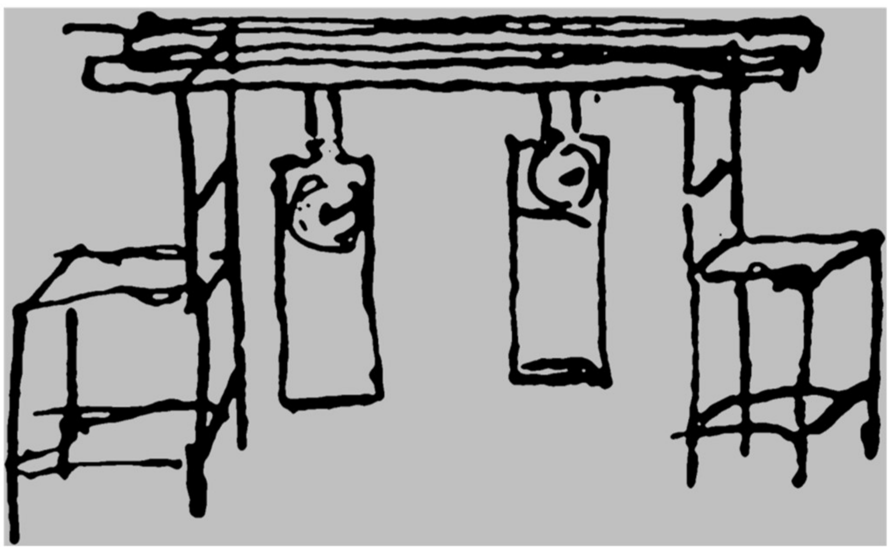
understanding interactions preview



synchronization

- Greek: *συν* (sýn), “together” and *χρόνος* (chrónos), “time”
- “share common time”

C. Huygens (1673)

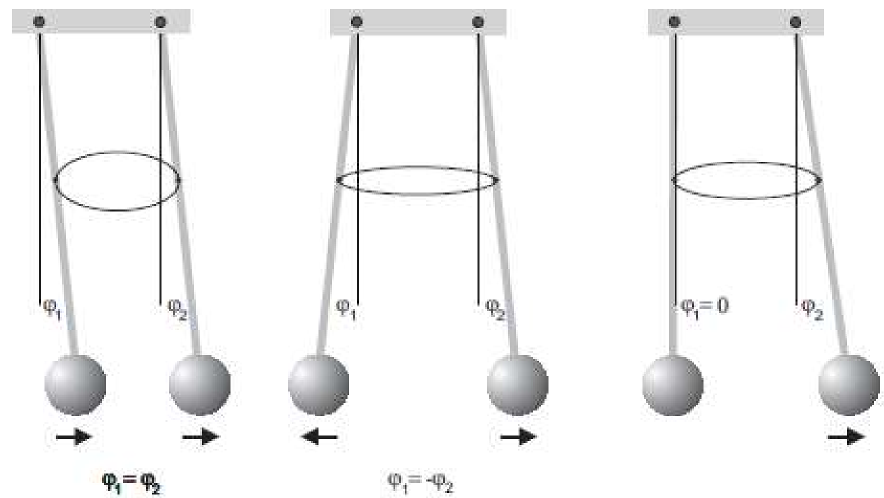


Two pendulum clocks hanging from a wooden support will transmit each other vibrations of very small intensity through the support. As a result, and this depending on other factors, the two clocks may start to oscillate at the same frequency, after some time

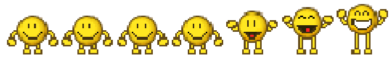
synchronization

paradigmatic model: coupled self-sustained oscillators

mechanics
electronics
laser physics
plasma physics
condensed matter physics
quantum mechanics
control theory
information theory
…



synchronization



fireflies

Kuala Selangor
firefly park (Malaysia)
www.fireflypark.com

J. Buck and E. Buck. Mechanism of rhythmic synchronous flashing of fireflies. Science, 159,1319, 1968.

shoal

neurons

from: physicsworld.com

flock of birds

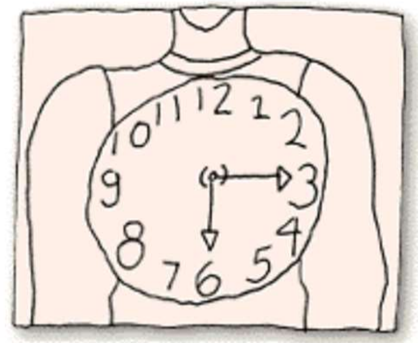
from: spiegel.de

clapping

Millennium Bridge London
synchronous lateral excitation

synchronization

biologic rhythms:



- ultradian (90 min.)
- circadian (24 h.)
- circaseptan (7 d.)
- infradian (28-32 d.)
- circannual (1 yr.)
- 7-years

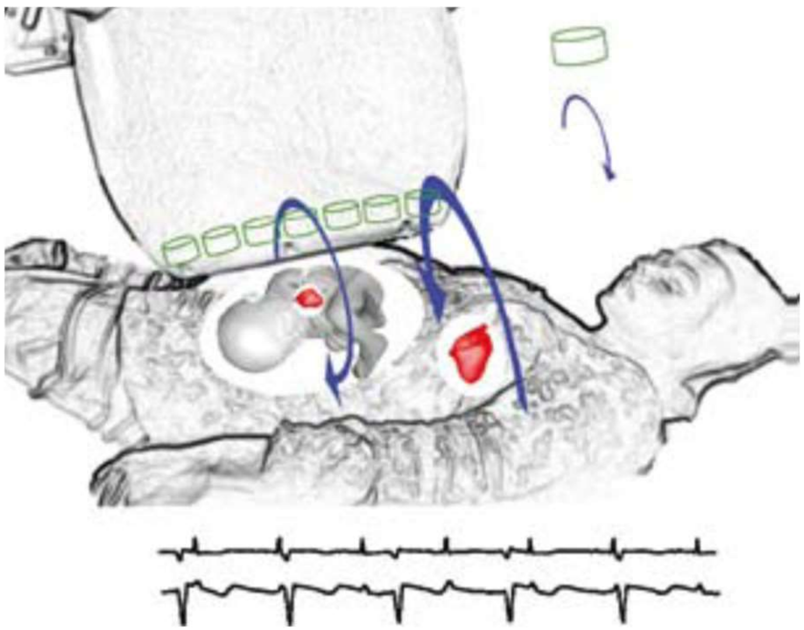
metabolism, food intake, sexuality, hormones, sleep, jet-lag, temper, ...

genes, proteins, ...

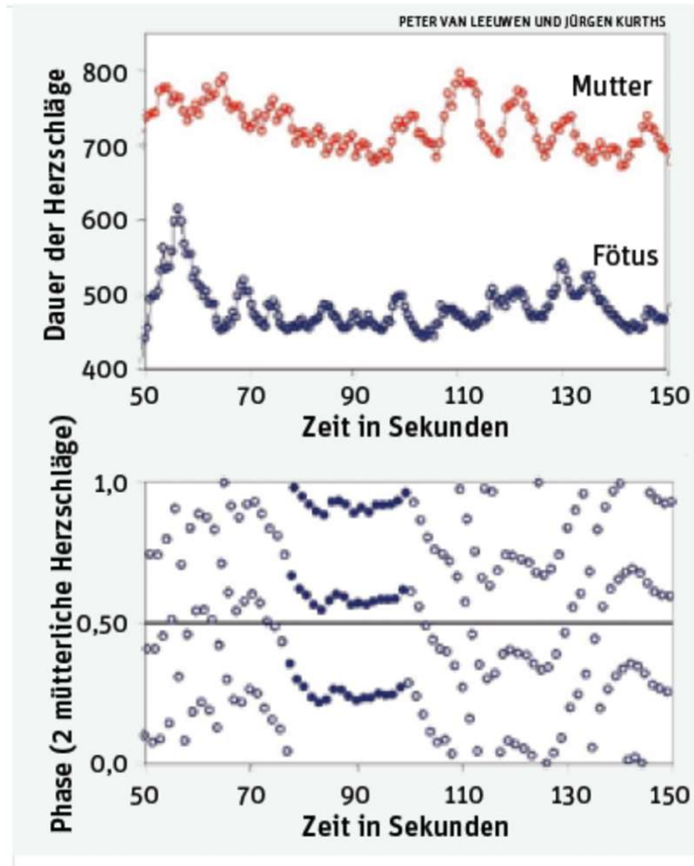
synchronization

hearts of mother and fetus in synchrony

- magnetocardiographic recordings of heart beats with superconductors (SQUIDs)
- synchronization analysis of heart beats



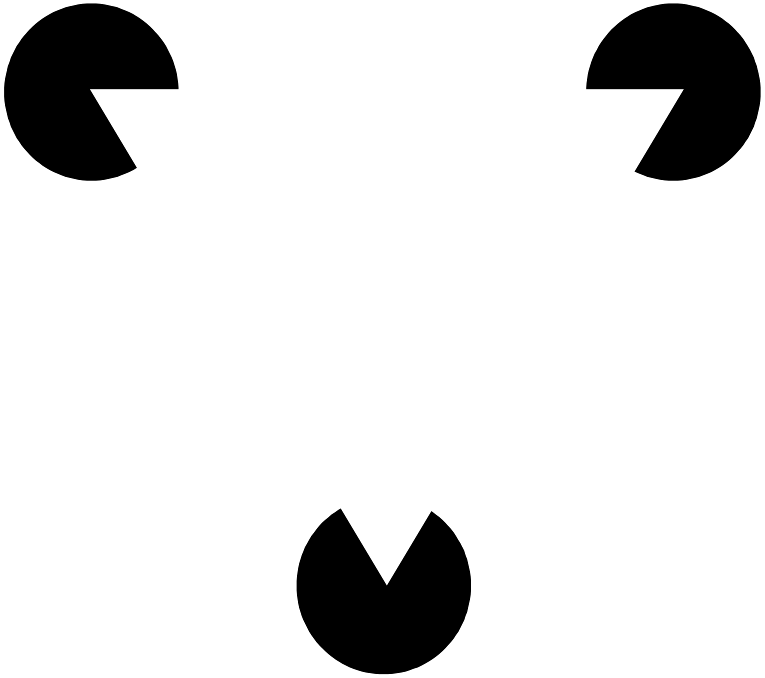
3:2 synchronization



Spektrum der Wissenschaft 22.01.2010

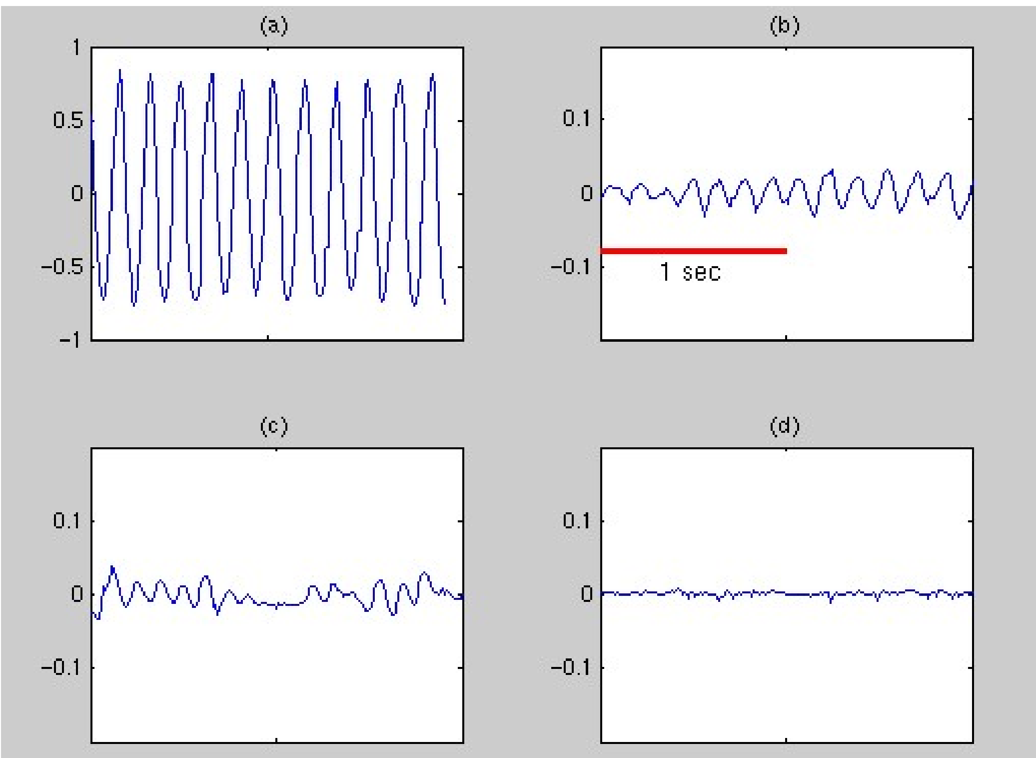
synchronization

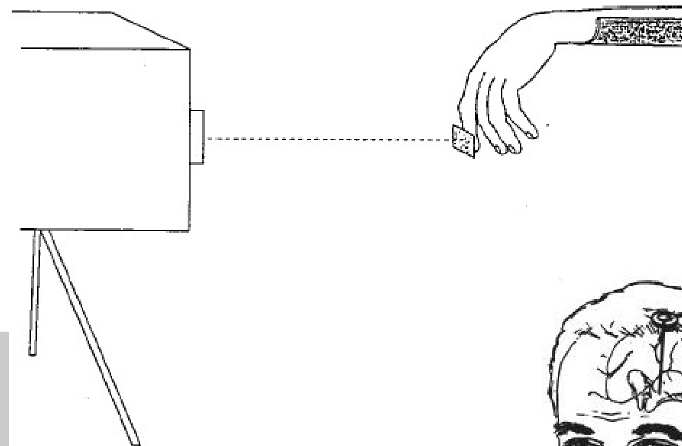
neurosciences: binding problem

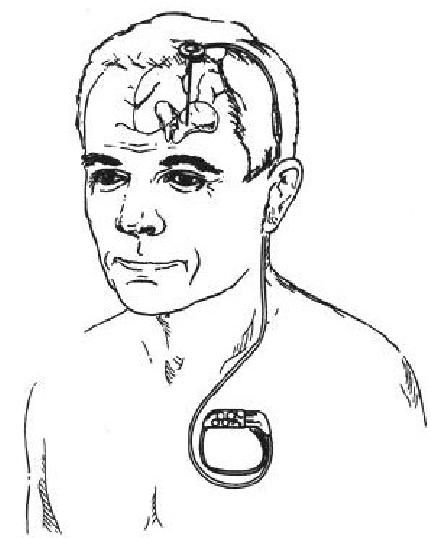


synchronization

Parkinson's disease





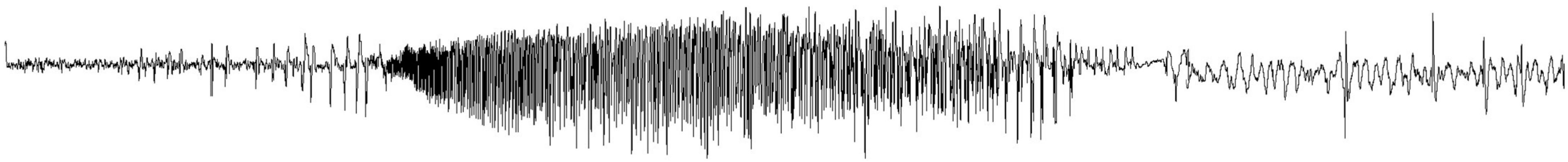


from: Medtronic Neurological

a) off
b) with counter stimulation / no meds
c) no stimulation / with meds
d) both

synchronization

epilepsy



from a highly
complex
irregular state

to a less complex
almost regular state
(synchrony)

back to a highly
complex
irregular state

synchronization

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Synchronization in Chaotic Systems

Louis M. Pecora and Thomas L. Carroll
Code 6341, Naval Research Laboratory, Washington, D.C. 20375
(Received 20 December 1989)

Certain subsystems of nonlinear, chaotic systems can be made to synchronize by linking them with common signals. The criterion for this is the sign of the sub-Lyapunov exponents. We apply these ideas to a real set of synchronizing chaotic circuits.

identical motion – despite chaotic behavior

synchronization

… a well-known but as yet not fully understood phenomenon

classical definition:

“adjustment of rhythms of oscillating objects due to a weak interaction”

extension for noisy *and* *chaotic systems:*

“induction of relationships between the functionals of two processes due to an interaction”

measuring interactions **analysis techniques**

- statistical techniques (cf. Linear Methods)

- information-theory-based techniques

- phase-based techniques

- state-space-based techniques

(- statistical-physics-based techniques (Kramers-Moyal expansion*))

* work in progress

measuring interactions statistical techniques

Consider two systems X and Y.

Given: time series $\boldsymbol{v}$: $v_1, v_2, \ldots, v_N$ of some observable $\mathbf{x}$ and
time series $\boldsymbol{w}$: $w_1, w_2, \ldots, w_N$ of some observable $\mathbf{y}$

Q:
how similar are time series $\boldsymbol{v}$ and $\boldsymbol{w}$?

Ansatz:
- consider difference time series $\boldsymbol{d} = \boldsymbol{v} - \boldsymbol{w}$
 (if necessary: appropriate normalization; e.g. zero mean, unit variance)
- assumption: time series similar up to some random component
- investigate moments of amplitude distribution of $\boldsymbol{d}$
- investigate autocorrelation of $\boldsymbol{d}$
- ….

measuring interactions **statistical techniques**

Consider two systems X and Y.

Given: time series $\boldsymbol{v}$: $v_1, v_2, \ldots, v_N$ of some observable $\mathbf{x}$ and
time series $\boldsymbol{w}$: $w_1, w_2, \ldots, w_N$ of some observable $\mathbf{y}$

Q:

how similar are time series $\boldsymbol{v}$ and $\boldsymbol{w}$?

Ansatz:

- compare amplitude distributions of $\boldsymbol{v}$ and $\boldsymbol{w}$
  (e.g., Student's t-test, F-test, Kolmogorov-Smirnov test)
- check for correlations between $\boldsymbol{v}$ and $\boldsymbol{w}$
  (Pearson's or Spearman's correlation coefficient)
- in case of strong delays: estimate cross-correlation between $\boldsymbol{v}$ and $\boldsymbol{w}$

measuring interactions **statistical techniques**

estimating cross-correlation brief recap

Pearson's correlation coefficient

$$\mathrm{cov}_{vw} := \frac{1}{N-1} \sum_{i=1}^{N} (v_i - \bar{v}) + (w_i - \bar{w})$$

$$r_{vw} := \frac{\mathrm{cov}_{vw}}{\sigma_v \sigma_w}$$

cross- correlation:

$$C_{vw^{\tau}} = r_{vw^{\tau}} \qquad C_{vw^{\tau}} = C_{wv^{-\tau}}$$

find delay that maximizes cross-correlation $\hat{\tau} = \mathrm{argmax}_{\tau} C_{vw^{\tau}}$

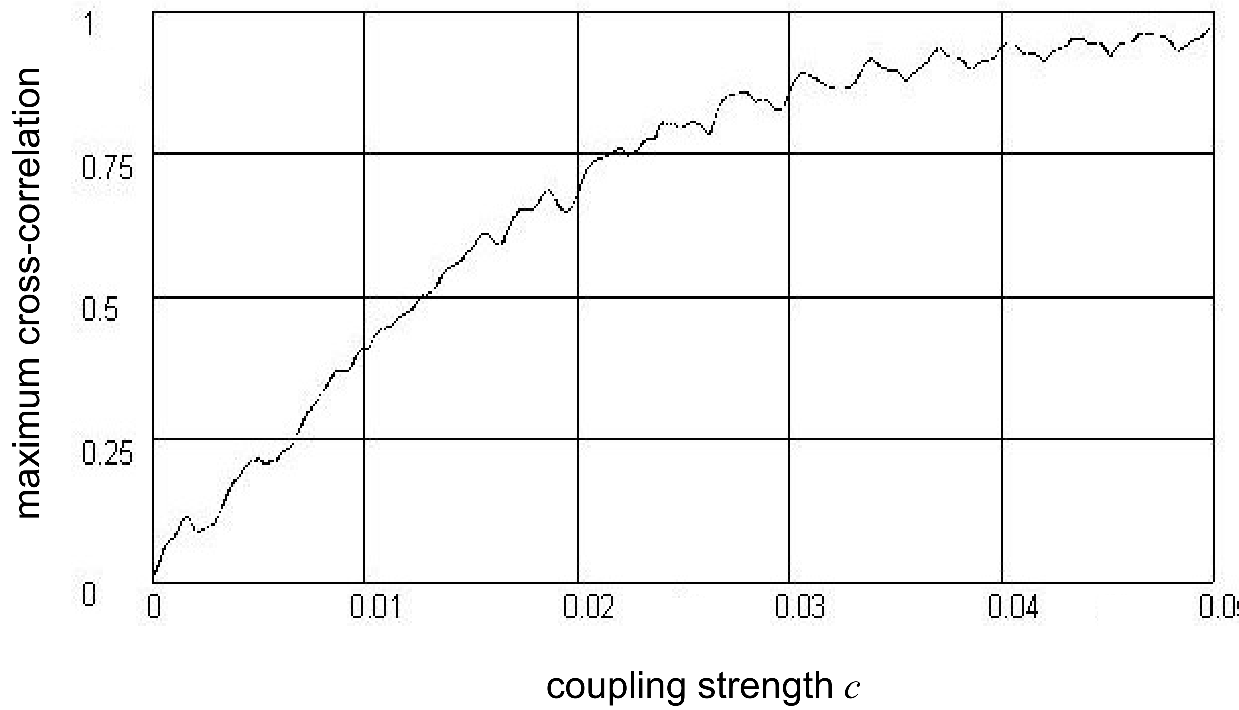
use maximized cross-correlation as measure for synchrony

measuring interactions **statistical techniques**

estimating cross-correlation examples

diffusively coupled oscillators (with eigen-frequencies Ω_v and Ω_w)

$$
\begin{aligned}
\ddot{\mathbf{v}} &= -\Omega_v \mathbf{v} + c(\mathbf{w} - \mathbf{v}) \\
\ddot{\mathbf{w}} &= -\Omega_w \mathbf{w} + c(\mathbf{v} - \mathbf{w})
\end{aligned}
$$

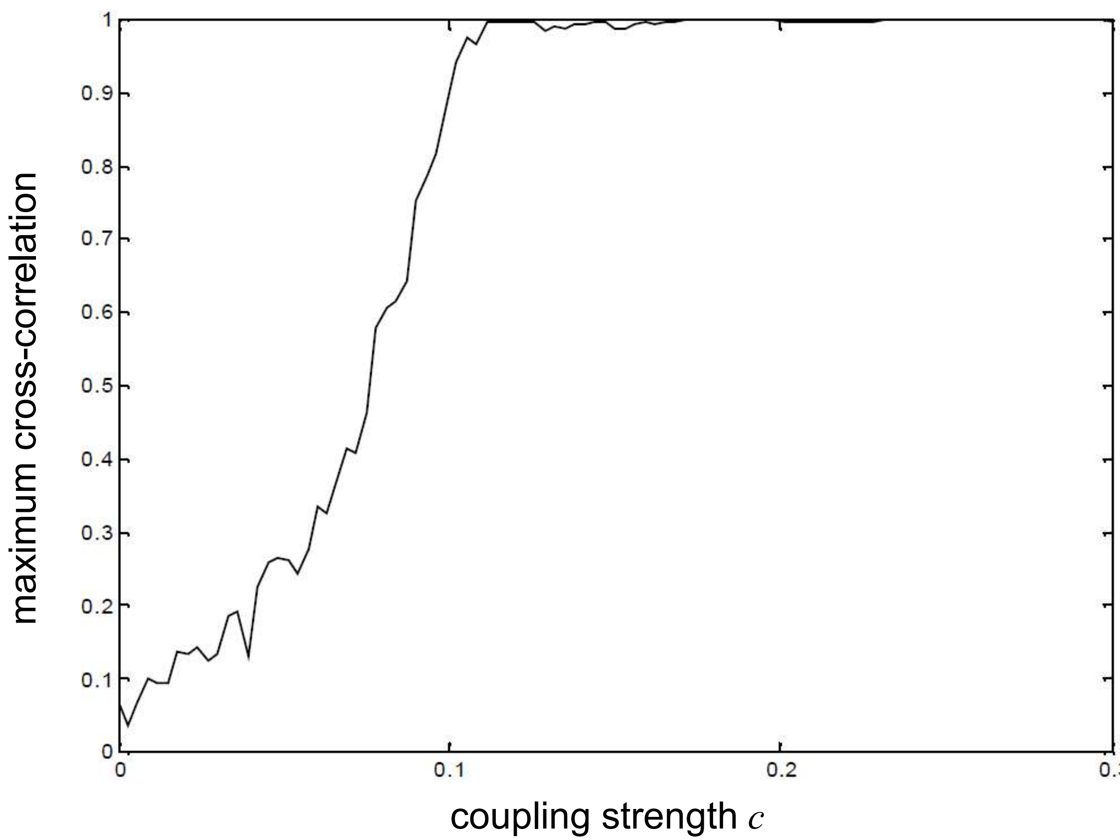


measuring interactions **statistical techniques**

estimating cross-correlation examples

diffusively coupled Rössler oscillators

$$
\begin{aligned}
\dot{x}_{1,2} &= -\Omega_{1,2} y_{1,2} - z_{1,2} + c(x_{2,1} - x_{1,2}) \\
\dot{y}_{1,2} &= \Omega_{1,2} x_{1,2} + 0.165 y_{1,2} \\
\dot{z}_{1,2} &= 0.2 + z_{1,2}(x_{1,2} - 10) \\
\Omega_1 &= 0.89;\ \Omega_2 = 0.85
\end{aligned}
$$

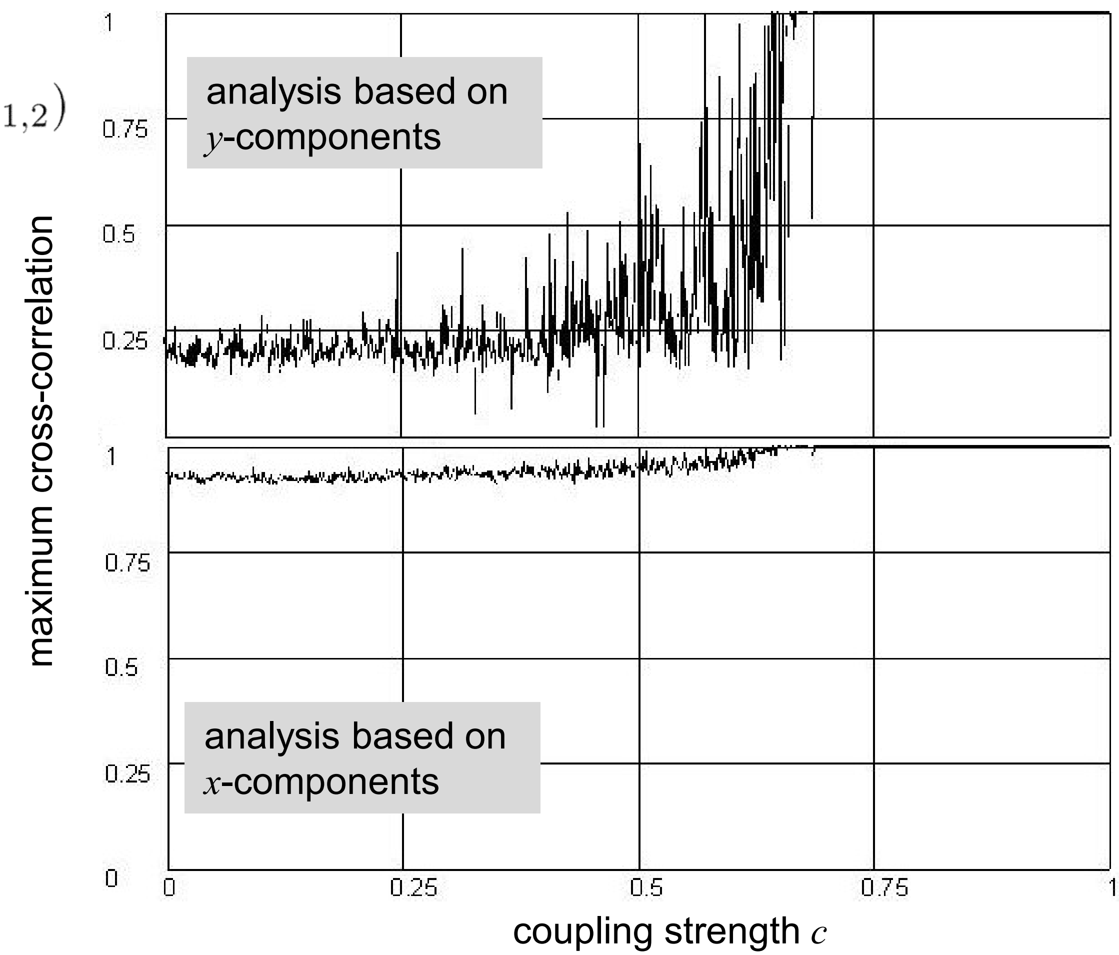


measuring interactions **statistical techniques**

estimating cross-correlation examples

diffusively coupled Lorenz oscillators

$$
\begin{aligned}
\dot{x}_{1,2} &= -\frac{8}{3}x_{1,2} + y_{1,2}z_{1,2} + c(x_{2,1} - x_{1,2}) \\
\dot{y}_{1,2} &= R_{1,2}z_{1,2} - y_{1,2} - x_{1,2}z_{1,2} \\
\dot{z}_{1,2} &= 10(y_{1,2} - z_{1,2}) \\
R_1 &= 28;\ R_2 = 28.00001
\end{aligned}
$$



measuring interactions **statistical techniques**

estimating cross-correlation in Fourier domain

cross-spectrum and coherence function

cross-spectrum is the Fourier transform $\mathcal{FT}$ of cross-covariance:

$$\Gamma_{\mathrm{vw}}(f) = \mathcal{FT}(\gamma_{\mathrm{vw}})(f) = \sum_{\delta=0}^{\infty} \gamma_{\mathrm{vw}}(\delta) \exp(-2\pi i \delta f)$$

$$\text{where } \gamma_{\mathrm{vw}}(\delta) = \mathrm{E}\left[(v(t) - \mu_{\mathrm{v}})(w(t+\delta) - \mu_{\mathrm{w}})\right]$$

$$\text{in polar coordinates: } \Gamma_{\mathrm{vw}}(f) = A_{\mathrm{vw}}(f) \exp i\phi_{\mathrm{vw}}(f)$$

(can also be estimated via Fourier transform of cross-correlation)

measuring interactions **statistical techniques**

estimating cross-correlation in Fourier domain

cross-spectrum and coherence function

coherence function (or magnitude-squared coherence) is defined as:

$$\kappa_{\mathrm{vw}} = \frac{|\Gamma_{\mathrm{vw}}(f)|^2}{\Gamma_{\mathrm{vv}}(f)\Gamma_{\mathrm{ww}}(f)}$$

it measures the frequency-resolved linear correlation between two signals

gives meaningful results for band-limited signals only

measuring interactions **statistical techniques**

general limitations

- see restrictions for Linear Methods
- analyses of amplitude distributions only
- do not (or only to some extend) provide information about joint dynamics

exception

- maximum cross-correlation increases monotonically with coupling strength → data-driven estimator for strength of interaction (at least for some cases); symmetric measure → no information about direction